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CHARLES DAY—SYMBOL OF AMERICAN INDUSTRIAL GENIUS *

BY

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In accepting the gracious invitation of the President and Board of Managers to appear before you and give the Charles Day Lecture, I was sensible alike to the honor and the responsibility conferred upon me. The invitation, and the opportunity it offered, made me most happy for it was my privilege to know Charles Day as a highly respected friend, especially after the year 1911 and up until his untimely passing on May 10th, 1931, at the early age of 52 years.

I can visualize him now with his delicate, refined, and keen features. I can feel the inspiring influences of his dynamic and orderly mind and his personal charm. Interesting scenes come to life vividly. I learned much from him and have not forgotten!

Over the years I have enjoyed the most pleasant relations with his very competent associates in the firm of Day & Zimmermann, Inc., for whom I have had the highest regard—John E. Zimmermann, Nicholas G. Roosevelt, Conrad Newton Lauer, William Findley Downs, H. R. Martz, and Charles Penrose.

Charles Day's early leaning towards engineering had the benefit of the great impetus to industrial and engineering progress furnished by the World's Fair at Chicago in 1893, shortly before he entered the University of Pennsylvania but when he was old enough to appreciate the desirability of an engineering career. He graduated from the University of Pennsylvania in the year 1899 and received his degree in Mechanical Engineering in 1903. His personal contact with the progress of electrical and mechanical engineering, through his work in

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1899 as technical consultant to the National Export Exposition held in Philadelphia, gave him a practical background of breadth internationally, superimposed upon his engineering training at the University.

He spent much of his summers, and for some time after his graduation, in the study of Scientific Management. He became fully informed of the pioneering work of Frederick Winslow Taylor and James Mapes Dodge and participated in the introduction of Scientific Management at the plants of the Link-Belt Company and the Midvale Steel Company. Under the firm name of Dodge & Day, Kern Dodge and Charles Day offered their services to the industry.

Three important factors entered as favoring the early work of the firm at the turn of the century: The new high speed machine tool steel was being introduced into American industry; and, at the same time, direct motor-drive in industrial operations was supplanting line shafting and pulleys with belt and rope drives. The third factor was that this same period saw the inception and growth of very large national corporate enterprises, then, many of them, erroneously designated as "Trusts."

However, the combination of the new processes and the extension and consolidation of many plants into larger properties brought with it a unique opportunity.

Scientific management started as a comprehensive and dynamic plan to reduce costs and improve the quality of products, through the introduction of better machinery and improved methods of handling and routing materials to and from the foundries and shops, and within the foundries and shops to and from machines. Among other things this led to scientific development of plant layouts and plant construction.

In 1911, Kern Dodge withdrew, and the firm became Day & Zimmermann, Inc. John E. Zimmermann had joined the firm in the year 1903.

The firm's activities broadened to include the management of public utilities, at the request of banking interests who believed that the methods developed in industry by Charles Day and his associates might be applied beneficially to light and power operations. Enlightened report work on mechanical fitness, operating problems, and financial structure of industries grew rapidly in volume; and the firm soon enjoyed an enviable position for the dependability of its reports and the soundness of its recommendations. These were the beginnings of contributions destined to attain international reputation.

One is impressed by the fact that when he enters the field of Scientific Management, his interest quickly broadens beyond the shops and foundries to include every phase of an enterprise; for any sort of waste, including wasted space, stands out like a "sore thumb" and immediately becomes the subject of an attack for improvement. Charles Day was insistent, with his associates that "bricks and mortar without earning power—are nothing worth."

If there is a good reason for improving one phase of an enterprise, there is no valid reason for neglecting others which should be improved. The efforts to cut costs and improve quality become a perpetual drive, with no let-up, against all elements which make up the total cost on which sales prices are based. In order that the efforts may be intelligently applied, strong ties are developed between production and accounting. Cost controls of production, adequate estimating, and budgeting are among the results.

One of the greatest, if not the greatest, contribution to the growth and strength of American Industry, has been this persistent, enduring, and scientific drive to improve. Tools and methods which are modern today will be discarded tomorrow if more efficient tools and methods are found. The drive is ruthless but very sound and in the interests of labor, management, capital, and the public. The competitive and profit motives, acting jointly in an unregimented and untrammeled American Free Enterprise System, have been a combination of intense forces for industrial progress not previously found in the World's history at any time or any where; and the end is not yet.

The assembly line in mass production plants; the Hoover Standardization and Economy Programs; the efforts of the Standards Committees of the great engineering societies, and of the industrial and business associations; the efforts of science laboratories, through research in both pure and applied science, which are bringing into use better materials and methods; and the Standard Test Codes and even Standard Contract Forms are all part of the outgrowth of the creative work of the early pioneers in Scientific Management, of whom Herbert Hoover was one.

There come to mind the strong ties between accounting and production costs and all that go with them, as exemplified in the extraordinary contributions of J. Howard Pew, who recently retired from the Presidency of the Sun Oil Company; of Ernest Robert Breech, now Executive Vice-President of the Ford Motor Company; and of Enders McClumpha Voorhees, Chairman of the Finance Committee and one of the top executives of the United States Steel Corporation.

I feel, Mr. President, that this notable gathering in The Franklin Institute, whose renown is a matter of great pride to all Philadelphians and to all Pennsylvanians and to men of science everywhere, offers a unique opportunity and a most appropriate occasion to present to you an analysis of *high morale*, that great motivating force behind progress and success. As a start I shall give the reason for my firm belief, based on personal experiences in many branches of human endeavor, that every industrial organization is made up of two broad and distinct but interdependent divisions:

First: The division which has most to do with the tangibles or the things which may be measured and expressed in quantitative units,

such as pounds, feet, square feet, cubic feet, pounds per square inch, dollars, minutes, etc., and

Second: The division which has most to do with the intangibles concerning which there are no quantitative units by which they may be measured or expressed and which arise out of the characters of enlightened men, such as the invaluable qualities of inspirational leadership; courage; enthusiasm; genuineness; alertness; vision; mutual helpfulness; freedom of expression; and goodwill, all of which are among the basic ingredients of that great composite quality—Morale. Whether high morale is present or not may easily determine the success or failure of any industrial organization, or a business enterprise, or a utility, or a union, or an association, or a team of athletes engaged in competitive sports.

Those who believe in and practice the qualities of high morale are almost invariably forward looking but at the same time are deeply interested in the past because vivid recollections of men and their works give an appreciation of the living present and sound visions of the future.

I have had the head of an important industry advise me that he did not believe in inspirational leadership, and he spoke as though he were letting me in on a secret. But I had for many months found the truth of his disbelief in the operations of his company. It was a pathetic situation, for he devoted his efforts exclusively, with great care and untiring energy, to the tangibles and to the perfection of corporate documentation, but suffered from acute color blindness in respect to the intangibles which may easily be, and usually are, more important. He was perfectly open and honest about the whole matter. His leadership did not last for long, for high morale was absent, and his visions of future trends were unsound. There was no rush to do business with his comapny. One could weep! And he is gone.

Charles Day not only was an expert in the field of the tangibles, but he preached and practiced also the qualities of high morale naturally and with wisdom and tact. He thus possessed a winning combination. In fact, I am of the opinion that one of his greatest personal assets was his ability to build morale. But he had something more in his make-up. It soon was plainly evident to me that he was a most responsible, courageous, and modest citizen; and the great respect in which he was held in the minds and hearts of his associates and legion of friends rested on his moral character. It is for these reasons that I consider Charles Day a true "Symbol of American Industrial Genius."

In a lecture such as this, I trust you will bear with me while I review some intimate personal experiences with *morale.* How high morale contributed to genuine progress and success. These experiences were had in a number of important and interesting fields—hydro-electric power; machinery building; the U.S. Navy Department in Washington

during the First World War; and believe it or not, in competitive sports. I also appreciate the converse of this, namely—a loss of morale may cause havoc and even quick disaster. So in pondering these thoughts I trust you will receive them kindly and then place yourself in my position and test yourself in accordance with your own experiences, and perhaps come up with far better thoughts. Apology must be offered for bringing myself prominently into the pictures; however, I believe you will agree that the lessons taught by actual experiences, selflessly presented, are usually far more valuable than those derived from theory alone.

In July 1905, one month after graduation from the University of Pennsylvania and at the tender age of 22 years, I was sent by William M. White, Hydraulic Engineer of I. P. Morris Company of Philadelphia, to Shawinigan Falls, Province of Quebec, Canada, to participate, as the Company's representative in the efficiency tests of a 10,500 horse power horizontal-shaft turbine of an improved type, designed and built for The Shawinigan Water and Power Company. I had just started in the drawing room of the Hydraulic Department. It was my first visit to Canada!

The I. P. Morris Company, founded in 1828, was a wholly-owned subsidiary of The William Cramp & Sons Ship & Engine Building Company, which operated a famous shipyard in the picturesque Kensington District of the City of Philadelphia, on the Delaware River. The shipyard was founded by my maternal great-grandfather, William Cramp, in the year 1830, at the age of 23.

It was on this trip when I first met John E. Aldred, a young banker from Boston, Wallace C. Johnson, Julian C. Smith, and John A. Walls, all of whom became great factors in the development of hydro-electric power in both the Dominion of Canada and the United States of America. With no advance notice, I found myself suddenly thrown in the company of distinguished men.

Wallace C. Johnson had been Chief Engineer of the Niagara Falls Hydraulic Power and Manufacturing Company—The Schoellkopf Development. He had branched out as a consulting engineer and, in the late 1890's, had an office in a little two-story wooden building at Niagara Falls, New York, located on the upper level of the gorge of the Niagara River, a short distance below the American Falls. One cannot conceive of more inspiring surroundings for an engineering office!

Julian C. Smith, Raymond D. Johnson, and John A. Walls were distinguished engineers and members of Wallace C. Johnson's staff.

Mr. Aldred knew of a great waterfall, the Shawinigan Falls of the St. Maurice River, wasting its energy in a wilderness in the Province of Quebec. He thought it should be developed and electric power be made available to the City of Montreal, 85 miles away. He was about to undertake a hazardous enterprise, for long distance electrical

transmission was in a rather primitive state. He visited the Schoellkopf Development to find out for himself the state of large-scale hydro-electric development and was so impressed by the character, vision, and ability of Wallace C. Johnson and his very competent staff, with what he had seen elsewhere, he engaged his services for the Shawinigan work.

When I arrived at Shawinigan Falls, in July 1905, the work was well on its way; and two earlier turbines of 6,000 horse-power each, built by the I. P. Morris Company from the designs of Escher Wyss & Company of Zurich, Switzerland, had been placed in operation as well as the 10,500 horse-power turbine already mentioned. With the advent of Mr. White, in 1902, as Hydraulic Engineer, the I. P. Morris Company broke completely away from dependence upon foreign designs.

I experienced two very startling surprises in working with the top members of the staff of The Shawinigan Water and Power Company; so pleasant, so unexpected, and so dramatic as a thought provoker, I can only say that it did much to shape my entire career.

First: I was treated as an equal which I did not deserve. However, as I was the official representative of my company, the representatives of the Power Company cooperated with me, in spite of my youth, with all of the courtesy and respect to which the position was entitled.

Second: I was perfectly amazed how those older men, uniformly and sincerely, practiced in all of their dealings among themselves and with outsiders, including myself, the intangibles of high morale which I have mentioned. After experiencing it day and night for a week's time and noting the benefits in the way these men worked as an enthusiastic team, I decided, then and there, that if ever I reached a position of high authority in the I. P. Morris Company, the very first thing I would do would be to see to it that these qualities were made a part of our industrial and public relations.

I did not have long to wait because, in 1911, Mr. White resigned his position as Hydraulic Engineer to become Manager and Chief Engineer of the Hydraulic Dept. of the Allis-Chalmers Mfg. Co. of Milwaukee; and after some delay, due to perfectly natural skepticism on the part of top management because of my youth, I was appointed Hydraulic Engineer and placed in charge of design, research, acceptance tests in the field, and sales. Thus Mr. White and I became competitors; a situation neither of us dreamed of just a few years before!

I took appropriate action to build morale immediately on assuming the position of Hydraulic Engineer. The results were immediate, electrifying and almost beyond belief! Morale arose to a high pitch. Everyone was made to feel that his efforts were important, and he became a tough and interested worker. The pace of the staff shifted to high gear. Creative talents, from which all advances arise, were encouraged; and broad and well-planned research programs instituted.

What good fortune we had in the outcome of our first job! We were well prepared; and having implicit faith in each other, we had the will to succeed. Associated with me were such outstanding young men as Lewis F. Moody, Frank H. Rogers, and R. E. Brunswick Sharp —all of whom become internationally known; and on the production side, John Overn, Jr., an older man broadly experienced in machinery construction and delightfully cooperative.

It was a relatively small contract, amounting to only $147,000, but its influence on hydraulic turbine design and powerhouse layouts was tremendous.

The contract involved seven turbines of a new vertical shaft single runner type (the runner of a turbine is the rotating element and in many installations there had been used, in the case of horizontal shaft turbines the complicated arrangements of two, four, or even six runners on a single shaft and for vertical shaft turbines as many as three runners on a single shaft). The seven new turbines were for installation in two powerhouses of the Appalachian Power Company on the New River in Virginia. All seven turbines were identical and the electric generator was placed directly above the turbine to form a simple, rugged two-bearing combined unit, and great care was exercised in the design of all water passages from the intake to the discharge in the tail race.

We earned a bonus of $40,000 for anticipating the guaranteed delivery schedule and a bonus of $60,900 for exceeding the guarantee of efficiency of 85 per cent by a World's Record of 93.7 per cent. (The maximum efficiency of the hydraulic turbine has advanced less than 1 per cent since that time (February 1913) or over a period of 39 years.) Thus we earned a total bonus of $100,900 on a $147,000 contract.

The personnel of the Hydraulic Department moved to as high a state of enthusiasm, loyalty, persistence, and cooperative action as I have experienced in any organization.

The conclusions reached were summed up as follows: It would be the dramatic start of a new era in which the vertical shaft single runner turbine, of great simplicity and rugged construction, would give the coupe de grace to all types of horizontal turbines and to the vertical multi-runner turbine; there would be greater accessibility and reliability; longer life; much lower maintenance costs; not to mention the lower first cost of hydro-electric plants. An evaluation of each of these advantages when compared with past practice, would show an appreciable saving; and when they were all added up, the sum constituted a major economic gain and the new turbine would find universal adoption on its merits.

As predicted, the vertical shaft single runner turbine came rapidly into use in both the United States and Canada, and then abroad. The installations which set the pace and which followed each other in rapid succession were: fifteen 10,000 horsepower turbines for the Mississippi River Power Company at Keokuk, Iowa; four 17,000

horsepower turbines for the Alabama Power Company, Coosa River, Alabama; twelve 10,800 horsepower turbines for the Cedars Rapids Manufacturing & Power Company, St. Lawrence River, Province of Quebec, Canada; and six 20,000 horsepower turbines for the Laurentide Company Ltd., Grand Mere, Province of Quebec, Canada.

Twenty-seven of those turbines were designed and built by the I. P. Morris Company and ten turbines by the Wellman-Seaver-Morgan Company of Cleveland, Ohio.

After these dramatic experiences, I was convinced that high morale in an organization can produce astonishing results, for all of the intangible qualities which make it up are very contagious and penetrate from the top to every extremity of an organization. Charles Day most certainly had the same concept. It was not surprising that I found in him a kindred spirit.

I shall never forget my obligation to the great men of Shawinigan Falls for their many kindnesses to me.

When Mr. Walls was a member of Wallace C. Johnson's consulting staff at Niagara Falls, he had been assigned in 1901 to make a layout and preliminary design of a power development on the Susquehanna River near Peach Bottom, Pennsylvania. Actual development along this reach of the river, a short distance from Peach Bottom, was initiated by the McCall Ferry Power Company, and five 13,500 horse-power vertical shaft two runner I. P. Morris turbines were completed for this power plant in the year 1910 and two 17,000 horsepower turbines of the same type in 1913. This was before the advent of the new vertical shaft single runner turbine.

In the year 1909, John E. Aldred had taken over the company and had organized the Pennsylvania Water & Power Company. Mr. Walls left The Shawinigan Water and Power Company to take charge of the engineering. As Mr. Walls was born in Lewisburg, Pennsylvania, he was returning near to the place of his birth on the banks of his beloved Susquehanna. He became Vice-President and Chief Engineer of the new company and then President, which position he held for many years and up to the time of his recent retirement.

The name McCall Ferry Dam was changed to Holtwood Dam. It is a most interesting and unique place today, with a large steam plant located alongside of the dam, the fuel for which is finely divided anthracite coal salvaged from the river. The Holtwood Dam is not far from Philadelphia.

Meantime, the concrete substructure of the powerhouse was altered, at considerable expense, for additional units, of the new vertical shaft single runner type, and the first turbine of 16,500 horsepower capacity was designed and built by I. P. Morris and installed in 1914. The wisdom displayed by the Power Company in assuming the considerable expense to accommodate the new type of turbine is evidenced by the

efficiency obtained, which was 93 per cent, or approximately 8 per cent higher than the turbines of the older design.

Mr. Walls and your speaker have remained close friends ever since we met at Shawinigan Falls in the year 1905. He always had been a brilliant engineer and an enlightened executive.

In August 1914, the First World War was well on its way. On May 8th, 1915, I was appointed Assistant to the President of the Cramp Company and the I. P. Morris Company. Henry S. Grove was the distinguished, broadly experienced and courageous President of both companies. My desk was placed in his office at his request. I was directed to think for myself and in the event of any differences between us in matters of policy, we were to work them out together so that our combined judgment would reach the Vice-President and General Manager, and the other officers and the department heads. I was to retain the active supervision of the Hydraulic Turbine business which by then had reached very substantial proportions.

Before long I sensed I would see much of Washington. I had very definite ideas regarding politics in matters so vital as those which concern the defense of our Country and I made up my mind that in any activities in which I may take part, politics would be out, in appearance as well as in fact, and I would be selfless.

Shipbuilding has been a "feast or a famine" business and required, among other things, for its successful administration: carefully prepared estimates as a basis for bids, which gave consideration to future trends; contracts carefully drawn so as to fairly protect the shipbuilder as well as the client; rigid control of work in process; conservation of funds in prosperous times to tide the company over lean years; and a constant effort to secure a diversification of product, which would fit into our highly skilled working force and splendid facilities, and thus tend to smooth out the dips in the over-all production curve.

Mr. Grove had been President since October 1st, 1903. Prior to his advent, the company had become involved in serious financial difficulties with a floating debt of some four and a quarter million dollars. A syndicate of public-spirited financiers of Philadelphia and New York, who desired to see the famous shipyard continued, advanced the money to pay off the floating debt by the issue of twenty-year serial notes. A majority of the stock was placed under a voting trust, with Mr. Edward T. Stotesbury and Mr. Richard H. Rushton of Philadelphia and Mr. George F. Baker of New York, the Voting Trustees. Mr. Grove was invited to sit on the Board of Directors. He was later made a member of the Executive Committee, then Chairman of the Committee, and then elected President. By the year 1915, the company under Mr. Grove's leadership, was restored to its former prosperity and influence.

Shortly after my appointment to the office of Assistant to the President, I received at my home a letter from Mr. Grove as follows:

Dear Birch:

You are on trial. If you come out with the respect of your colleagues you will have made a great step.

Keep your head. Be loyal to the company. Don't be misled by sophistry.

Sincerely,

/s/

Henry S. Grove

I have always considered this letter one of my gems; for in just a few lines it offers so much constructive philosophy! Whenever I have gotten into a tight spot the words appear before my eyes. Many times it has been most helpful in arriving at a safe and sound course of action.

As expected, I became increasingly active with the Navy Department in Washington. Late in the year 1916, I attended a number of meetings, on consecutive Sundays, called by the Assistant Secretary of the Navy, The Honorable Franklin D. Roosevelt. The meetings were held in his Washington home and were attended by a number of shipbuilders and distingushed Naval officers. Mr. Louis McHenry Howe, the friend and assistant of Mr. Roosevelt, was there and so was Paymaster George P. Auld, a top-notch accountant whom I met again in Paris in 1920, when he was Accountant-General of the Inter-allied Reparation Commission. Mr. Roosevelt had taken the lead in the development of the preparedness program. We discussed the various aspects of shipbuilding; the probable cost of Battle Cruisers and the methods of determining their cost which would take into consideration the fact that a considerable proportion of the materials would be negotiated directly by the Navy.

After each conference Mrs. Roosevelt, who was a delightful hostess, served tea and the youngest of the Roosevelt children would have great fun playing "horse and rider" with Mr. Howe, who would get down on his hands and knees and give the youngster a ride on his back.

Then during the days from January 15th to the 17th inclusive, 1917, the Committee on Naval Affairs, House of Representatives, Honorable Lemuel P. Padgett, Chairman, held hearings on the Cost of Preparedness Program, before which many Naval officers and shipbuilding executives testified. The Cramp representatives—Mr. Henry S. Grove, Mr. J. H. Mull, Manager of the Marine Department, and your speaker—testified on the 17th. Much of the time was devoted to Battle Cruisers, a new type of vessel, in which the Cramp Company was not especially interested, because it was too large for our then existing facilities.

It was quite evident to me, from the testimony, that the Navy was leaning toward a cost plus percentage type of contract, and the shipbuilders were strongly in favor of it, for costs had been jumping by

leaps and bounds. Because of the great risk involved, no prudent management could afford to take work on a fixed price basis. The problem seemed to be to find an adequate definition of costs which would have legal standing. I felt that the method of determining the cost of changes to a ship, namely: actual labor and material, plus 10 per cent of material, plus 50 per cent on labor, was a definition entirely inadequate for determining the cost of a complete ship, although it had been recited in Naval contracts for years and therefore had a legal standing. However, it seemed to me that a comprehensive definition of cost could and should be devised and then made legal by an Act of Congress.

On January 25th, 1917, I was elected a Director and Second Vice-President of the Cramp Company and, at a meeting of the Board of Directors, held on February 17th, all Washington transactions were placed in my hands.

While in Washington I saw quite a little of my good friend Joseph W. Powell, President of the Fall River Shipbuilding Corporation. Mr. Powell had been a Naval Officer but had resigned to become Assistant to the President of the Cramp Company in 1906, where he remained until 1914, when he resigned to become President of Fall River. I had succeeded him as Assistant to the President of Cramp, in 1915. He was a spendid shipbuilder, an attractive person of high integrity, and blessed by a great reservoir of energy.

I also met for the first time, Homer L. Ferguson, President of the Newport News Shipbuilding and Dry Dock Company, and had quite a number of meetings with him and members of his staff. He also had started out as a Naval Officer. He was the most attractive of men, a great patriot, and a splendid speaker. His Southern accent and his black eyes with an unconscious show of emotions, whether in a serious or jovial mood, made him most popular. He taught a valuable lesson —"Be yourself." We worked together in an effort to secure a sensible cost plus a percentage form of contract which we desired to have the Navy adopt.

Mr. Ferguson called me on the telephone to advise that he had discovered a new legal definition of cost in the first Munitions Tax Law. This definition became the foundation of the new Navy cost plus a percentage form of contract.

About this time, I had several meetings with the representatives of the other shipbuilding companies. The Honorable Josephus Daniels, Secretary of the Navy, had requested the shipbuilders to form an association so he could meet their representatives in a group and thus save the time that so many individual meetings would require. The Atlantic Coast Shipbuilders Association was the result. I became active in its affairs and headed several important committees.

Then I saw much of Admiral David W. Taylor, Chief of the Bureau of Construction and Repair and one of the greatest men I ever had the pleasure of meeting. I wish I could claim relationship! He was a

Godsend to Secretary Daniels, who leaned on him more and more as time went on and with justification.

I was present at a splendid demonstration of confidence the Secretary placed in Admiral Taylor. I had taken to the Admiral a project on behalf of the Cramp Company. The Admiral thought it was important and that we should see the Secretary about it. When we arrived at the Secretary's office, Admiral Taylor explained the project to Mr. Daniels and Mr. Daniels said something like this to the Admiral:

"Admiral Taylor, you are the most distinguished Naval Constructor in this Country and perhaps in the world; you are held in the highest esteem by the Navy for your ability and integrity and you have a good business head. Anything you two Taylors agree to I will approve."

This was the first time I ever saw the Secretary pass authority on to another. Things began to move.

I have in my possession a gem in hindsight from Mr. Daniels regarding Admiral Taylor! I received a letter from him dated August 8, 1947, just about thirty years after our meeting in Washington and about five months before his death on January 15, 1948, in his 86th year. It said in part:

"Admiral Taylor was the most many-sided man and knew more about many things than any man in the Navy during the years I was connected with it. He was the frankest and most helpful adviser in the eight years I was in Washington. It was tragic that in his last years he was physically out of commission and had to remain in the hospital and his mind was as clear and his spirit as wonderful as when in the prime of his health."

On February 10th, 1917, I called on Mr. Roosevelt and suggested that in order to save time for him it might not be a bad plan to request the Newport News Company and the Cramp Company to prepare jointly an Emergency Repair Contract, and submit it to him for criticism. He thought well of the idea and requested both companies to proceed with the work.

On February 19th, we met with the shipbuilders and the representatives of the repair yards and went over the final form of the proposed contract before submitting it to Mr. Roosevelt. On February 23rd, at at the call of Mr. Roosevelt, all of us met in his office. Several paragraphs had been revised and submitted to all concerned for approval. There was a meeting of the minds and the approval permitted the immediate conditioning and repair of Naval vessels.

On March 6th, I attended a meeting of the shipbuilders in the office of the Secretary of the Navy, followed by a meeting in Mr. Roosevelt's office. These conferences had to do with the award of contracts for 6 Scout Cruisers and 5 Battle Cruisers.

On March 14th, the bids for Scout Cruisers were opened. Cramp was the low bidder at $5,950,000 each for two. The General Manager, the Chief Engineer, and the Naval Architect of the Cramp Company arrived in Washington for the opening of the bids. I was amazed

when the General Manager, in a surprise move, took the position that the Cramp Company preferred a fixed price contract to a cost plus a percentage contract, because "it was so much easier to administer!" This was indeed a surprise when it is considered that Mr. Ferguson and I had for some time been strongly advocating the cost plus a percentage contract, as the only just one in an uncertain market, and did our best to so convince the Secretary.

Thus Cramp was awarded contracts for 2 Scout Cruisers amounting to $11,900,000 at a fixed price in a rapidly rising market! I found that the General Manager had figured he had a margin of 20 per cent over the cost and therefore it was more desirable than a cost plus 10 per cent contract. He seemed to believe that the 20 per cent margin would be profit and gave no consideration to the prospect of rising costs during construction wiping out this margin. Admiral Taylor advised me that pressure was put on him by the Cramp people to have the fixed price contract accepted! It was beyond my understanding why anyone would want to take such a risk.

As a matter of interest let us look back and see what the actual advances were in the costs of labor and material in the three years 1916–1919. Labor advanced 178 per cent and material 100 per cent. Do you not agree that this was no time for fixed price contracts unless one wanted to gamble with the odds strongly in favor of crippling losses?

On March 24th, the Secretary awarded to Cramp a contract for six Destroyers on a cost plus a percentage basis.

On April 6, 1917 President Wilson issued a proclamation that "a state of war exists between the United States and the Imperial German Government."

On June 17th, I called on Admiral Taylor in regard to additional Destroyers the Navy required. The Navy wanted 100 more Destroyers and which were to have priority over everything else. I pointed out to Admiral Taylor and the Secretary that it would not be fair to postpone the work on the two Scout Cruisers which were taken on a fixed price basis, in a rising market, and give priority to Destroyers which were contracted for on a cost plus a percentage basis. The Secretary agreed and approved a change in the contract for the two Scout Cruisers to a cost plus percentage basis. What a relief that we need not suffer the consequences of an inexcusable error in judgment!

At the time of the execution of the contract the 20 per cent margin over cost which the General Manager believed he had in the Cruiser contract was more than wiped out by rising costs, and the work had hardly started.

Then, on July 20th, Admiral Taylor advised me that the Secretary had wired the Cramp Company that it had been awarded a contract for 15 Destroyers on a cost plus percentage basis.

On August 9th, I contacted Admiral Taylor who said that the Navy would require still more Destroyers, in fact 150, and they must be delivered at the same time as the last lot of 100 (of which 15 had been

awarded to the Cramp Company). He wanted to know promptly what Cramp could do in the matter.

It seemed to me that it would be necessary to acquire, at once, additional existing production facilities in order to do our share.

I was confident the Navy would not award contracts for additional Destroyers to Cramp, or any other shipbuilder, without proof that they could be turned out.

On September 10th, 1917 I went to see Admiral Taylor and Admiral Griffin, Chief of the Bureau of Steam Engineering, and offered to take 25 additional Destroyers valued at about 37 million dollars, provided the Secretary of the Navy would approve certain suggestions regarding an increase in production facilities as follows:

1: That the Secretary would purchase the capital stock of the De La Vergne Machine Company which manufactured oil engines and refrigeration machinery and had a fine plant in New York City. It was a German concern with a highly skilled working force and splendid shop facilities. We had heard rumors that the Alien Custodian was about to take over the company. We believed that we could turn out in this plant the steam turbine propelling machinery for the 25 Destroyers. In answer to the question as to the price which should be paid for the stock, I recommended that Day & Zimmermann, Inc., a most reliable concern, be engaged by the Secretary for the purpose of determining a fair value of the De La Vergne Machine Company.

2: That the plant of the Wellman-Seaver-Morgan Company in Akron, Ohio, be enlarged to manufacture the condensers, uptakes, and stacks for the 25 Destroyers.

On September 25th, I visited the Secretary of the Navy and he made the following preliminary awards:

25 Destroyers to Cramp
14 Destroyers to New York Shipbuilding
20 Destroyers to Newport News

The Secretary approved the plan in regard to the De La Vergne Machine Company, with Day & Zimmermann, Inc., to make the appraisal, which I considered a tribute to the integrity of that company; and the Cramp Company to operate the plant as an agent of the Navy. He also approved the plan for the expansion and use of the Wellman-Seaver-Morgan Plant but modified so that it would manufacture the condensers, uptakes, and stacks for the Destroyers of the three companies, instead of for the Cramp Destroyers alone.

On December 5th, 1917, I sent a letter to The Honorable Edward N. Hurley, President of the United States Shipping Board Emergency Fleet Corporation, advising him that Destroyers had been given priority over everything else and the merchant vessels we were building for the Ward Line, Grace Line, and for others, and which had been commande-

ered, would be delayed and as a matter of justice the contracts should be changed to a cost plus a percentage basis. This was approved.

All of our contracts were now in good shape and I felt greatly relieved.

It was about this time when a most pleasing incident occurred out of the blue!

A gentleman stopped me on Pennsylvania Avenue near The White House and said, "How do you do, Mr. Taylor. Do you remember me?" When I hesitated for a moment he added: "I am Lemuel P. Padgett, Chairman of the Committee on Naval Affairs, House of Representatives." I then quickly recalled that I had testified before the Committee on January 17th in regard to the Cost of Preparedness Program.

Mr. Padgett then said: "Mr. Taylor, you are a Republican and we here in Washington are mostly Democrats, but I want to say to you that no one has cooperated with the Navy better than you have and no one has treated the Navy fairer than you have and I hear this from all sides." I was overwhelmed!

I thanked Mr. Padgett profusely and after discussing the shipbuilding situation we parted. I was so taken back by this generous and unexpected compliment, I walked on thin air for several blocks until I came down to earth. My determination to keep out of politics in appearance, as well as in fact, seemed to have been of great help.

Shortly after President Wilson issued his proclamation that "a state of war exists between the United States and the Imperial German Government," Charles Day offered his services to his Country and they were immediately accepted. He served with Charles Piez and Arthur Mason on a committee to report on the condition of the shipbuilding plants in the United States. He accompanied the House Mission to England and France in the fall of 1917 as a special representative of the Shipping Board when he gathered much information relative to the tonnage contribution that we would have to make to meet the requirements of the various military programs and the probable losses through the enemy's submarine campaign. He secured reliable information concerning the losses due to submarine warfare, British and French shipping and shipbuilding data.

Early in 1918, he returned to the United States and was made a member of the Army War Council and Trustee of the United States Shipping Board Emergency Fleet Corporation. In the summer of 1918 he again went to France and England, this time with E. R. Stettinius as a special representative of Secretary of War Baker and Mr. Hurley of the Shipping Board. He visited all ports, storage and terminal facilities, and lines of communications used by the American forces and reported on the adequacy of these facilities. He took part in numerous conferences with General Pershing and the British Ministry of Shipping, relative to such problems and, cooperating with Dwight W. Morrow, assisted in securing British tonnage for the United States

Army movement and participated in the settlement of the controversy regarding the payment to Great Britain for moving American troops.

On November 11th, 1918, the Armistice with Germany had gone into effect.

On June 26th, 1919, I forwarded a proposal to the Secretary of the Navy in which the Cramp Company offered to purchase the stock of the De La Vergne Machine Company for the same price the Navy had paid for it. After the exchange of letters to clean up details the purchase was consummated. The price paid was $1,275,000. This was part of the plan to diversify the operations of the Cramp Company. The purchase brought to us two new lines of work—Refrigeration Machinery and Diesel Engines. When the transaction was closed the Secretary of the Navy made the remark that he believed it was the first time the Navy had received 100 cents on the dollar for any facility provided by the Navy for the shipbuilders.

Shortly after the Armistice, the Canadian power interests had decided that Canada had reached the position where the requirements for hydraulic turbines, paper-making machinery and other heavy machinery, were such that they could be economically and entirely built in Canada and thus save the duty charge which Canada had paid for years on the importation of this machinery from the United States and Europe and give employment to Canadians.

I recalled the statement of Sir Wilfrid Laurier (the Premier of Canada at the turn of the Century)—"The 20th Century belongs to Canada."

The Dominion Engineering Works, Ltd., of Montreal was organized in May 1920. It agreed to enter into a contract with Cramp (I. P. Morris had become a department of Cramp) for the exclusive use in Canada and the British Empire for its designs of hydraulic turbines and other hydraulic machinery for which the Cramp Company would receive certain fees and a share of the earnings. The contract was to run for a period of 10 years.

Sir Herbert Holt, President of the Montreal Light Heat & Power Company; George Chahoon, Jr., President of The Laurentide Company Ltd; Julian C. Smith, General Manager and Chief Engineer of the Shawinigan Water and Power Company; Howard Murray, Vice-President of Aldred & Company, and your speaker agreed to serve on the Board.

This company today is one of the most important designers and builders of hydraulic turbines in the world. It also builds Paper Machines, Diesel Engines, Diesel Locomotives and material handling machinery. It has been most successful. It has earned a splendid reputation for the high calibre of its work and splendid service to its customers.

The I. P. Morris Company has built practically all of the turbines operating in the various plants on the New York side of the Niagara River and a substantial part of those operating on the Canadian Side.

During the early period from 1895 to 1900 it built 10 vertical shaft turbines of 5000 horsepower each, from the designs of Faesch and Piccard of Geneva, Switzerland for Station number 1 of the Niagara Falls Power Company, head 136 feet; and from 1902 to 1904, 11 vertical shaft turbines of 5500 horsepower each from the designs of Escher Wyss & Company of Zurich, Switzerland for Station number 2 of the same company, head 148 feet.

In 1901, it designed and built 5 horizontal shaft turbines of 2900 horsepower each and 4 horizontal shaft turbines of 2300 horsepower each for the Niagara Falls Hydraulic Power & Manufacturing Company (the Schoellkopf Development), head 210 feet.

Then, from 1904 to 1914, it designed and built 13 turbines of 10,000 horsepower each for Station Number 3 of the same company, subquently called the Hydraulic Power Company, head 210 feet. These turbines, which were started under the direction of William M. White before he resigned, were about the last word in design and efficiency for turbines of the horizontal shaft type—efficiency 90.3 per cent.

These latter turbines were put in under the direction of John L. Harper, Vice-President and Chief Engineer of the Hydraulic Power Company. He was one of the smartest engineering administrators I ever had the pleasure of knowing. He practiced the qualities of high morale. He had splendid vision and combined great courage with gentleness of manner; a man of fair and candid mind and straight forwardly realistic; a genius in drawing out the very best in those who worked for him, he gathered around him a brilliant engineering team and to it he gave all credit. I was deeply impressed with his constant drive to get the greatest amount of electrical power out of the restricted amount of water allotted to his company. We became very close friends.

Then following the advent of the new vertical shaft single runner turbine (the Schoellkopf Development had always used horizontal shaft turbines) Mr. Harper switched to the new type of vertical shaft turbine and placed contracts for 3 turbines of 37,500 horsepower each. Two turbines by I. P. Morris and 1 turbine by Allis Chalmers.. Year of completion 1919—efficiency 93 per cent, head 213.5 feet.

The corporate name of the Schoellkopf Development changed from time to time, then finally to the Niagara Falls Power Company as various consolidations took place to bring the water under the lower heads of the earlier plants (136 feet and 148 feet) to the higher heads available at the Schoellkopf Development (210 feet and 215 feet), and utilized in far more efficient machinery.

Then three turbines of 70,000 horsepower each were ordered. Two turbines by I. P. Morris and one turbine by Allis Chalmers—efficiency 93.8 per cent, head 215 feet. These turbines were the most powerful and most efficient in the world; and these records they held for many years.

In the autumn of 1923 Mr. Harper invited me, with the backing of the Schoellkopf interests, to form a partnership; and Harper & Taylor,

Incorporated, was organized in January 1924. I was to retain my connection with Cramp and Mr. Harper with Niagara Power. We took over most of the engineering staff of the Niagara Falls Power Company and we also opened an office in Philadelphia. Mr. Harper advised me that some of the partners of Day & Zimmermann, Inc., had conferred with him regarding the possibility of combining with Harper & Taylor, Inc., and that he was much impressed with the high calibre and broad experience of the Day & Zimmermann men. We decided to give this idea full consideration a little later.

The whole plan was completely upset by the sudden and lamented death of Mr. Harper, on November 28th, just a few months after we were organized. It was a great shock to me to lose such a close friend.

Harper & Taylor was terminated and the subscriptions refunded.

If all those responsible for the direction of competitive sports believed in and practiced the qualities which build high *morale*, the recent rising tide of distressing attacks on the conduct of competitive sports would not have taken place. But there were some sections in the great world of sports which failed to practice these qualities and were, therefore, vulnerable to attack. Unfortunately, and there's the rub, some of these attacks were not discriminating and tended to injure what is good as well as to correct what is evil.

Evils are not eliminated by dealing with symptoms and effects instead of basic causes. Still stands the fact that the choice of our actions is moral.

When morale is high:

First: There are no incentives to win by any means fair or foul! All abuses and perverted methods such as cheating, bribing, and sly infractions of the rules, which endeavor by unfair means to make losers out of rightful winners and winners out of rightful losers, do not arise.

Second: The passion to win is there, as it should be, but not merely in accordance with the technical rules of the game, as devised by the Committees on Rules, but with that more exacting code which demands adherence to the tenets of enlightened sportsmanship at its highest.

Third: No matter how keen the rivalry, good sportsmen repudiate with abhorrence foul tactics such as injuries purposely inflicted to win.

Fourth: The students have no desire to be diverted from their lessons. They stand by their obligations with great persistence and are faithful to the training rules. Secret violations seldom occur.

When a team loses, but has tried its best, it deserves the unstinted praise and support of the students, the alumni, and the public. Good sportsmanship on the part of those gathered in the stands is so important and of great help in toning up the game.

I know football, have played it, and have done some coaching. I have followed the careers of many players. I consider football the King of Sports and when properly conducted and inspired is a great character builder. Those who have graduated from football and thus

have had the experiences of playing on the "fields of battle" and in the public eye, are almost invariably men of good character and of considerable influence.

When men are properly conditioned and trained, by methods suited to the exacting requirements of the game, they play with joy in their hearts, for the game has become real fun. They are not short of breath and their muscles are toughened to withstand shock. Injuries are reduced to a minimum.

There is much virtue in spring practice when it aids young men to keep in good condition and with a definite object in view. Condition means so much to the success and pleasure of a game so exacting as football.

John H. Minds, one of Pennsylvania's greatest football players and my ideal of a sterling sportsman, All American and Captain of the undefeated 1897 team (a light team), who played on four teams 1894–1897 inclusive, which won 55 games and lost only one, and the first player to execute a placement kick in football history (against Harvard in 1897), tells me that George Woodruff, the famous creative coach during his time, had this to say about the combination of the three phases of college life; scholastic responsibilities, athletics and social activities:

> A college man who desires to stand well scholastically, could take an active interest in athletics, or in social activities, but he could not succeed scholastically if he tried to combine all three phases of college life. Such an effort would cause so much distraction that the student could not concentrate on any one of the three. Anything worth doing is worth doing well and one should apply his efforts accordingly.

I indulge in the thought that a man should give up social activities for the short time he is devoting his energies to the combination of his lessons and athletics.

Dr. Carl S. Williams and William M. ("Big Bill") Hollenback, two famous Captains and Coaches and close friends of mine for many years, subscribe to the thoughts of George Woodruff.

Although the technics of the game of football have changed over the years and will probably change in the future, the tenets of good sportsmanship never!

Just before I was elected President of the General Alumni Society of the University of Pennsylvania, on June 6th, 1930, I had heard rumors that Thomas Sovereign Gates, the distinguished banker and citizen of Philadelphia, had been persuaded by Charles Day, who headed a small group of influential citizens, to give up his banking connections and accept the nomination for the office of President of the University of Pennsylvania.

One of the prominent citizens who took part with Charles Day in this splendid move was my good friend Colonel Samuel Price Wetherill, a leader in civic affairs of Philadelphia. Colonel Wetherill was President of the Reginal Planning Federation of the Philadelphia Tri-State District. The Board of Directors of the Federation was made up of a

large number of leading citizens and with a Technical Advisory Committee of nearly 200 picked citizens, representing Philadelphia and surrounding districts in Pennsylvania, New Jersey, and Delaware. Thomas Sovereign Gates, Charles Day, and Colonel Samuel Price Wetherill were very active in the Federation and in the affairs of the University of Pennsylvania.

Colonel Wetherill said this to me regarding Thomas Sovereign Gates and Charles Day:

> One of my happy memories was a conference we had in our mutual hope that the demonstrated ability of Thomas Sovereign Gates should be devoted entirely to civic and educational work, while he still retained his vigor and leadership ability.
>
> At this late date it seems no breach of confidence to recall our conferences at which Charles Day persuaded Mr. Gates to accept nomination to become President of the University of Pennsylvania.
>
> Mr. Day's excellent judgment was tempered by a high sense of public responsibility and these characteristics were contagious and inspired all of us who were planning the best development of the Region. In addition to business astuteness and personal charm, he had a fine appreciation of artistic and ethical values. I count it a high privilege to have known and worked with him.

Thomas Soverign Gates became the *first* President of the University of Pennsylvania, in July 1930 (a new office), and it was my pleasure to see him in action during the four terms I was President of the General Alumni Society and the ten years I was a Trustee of the University.

I know of no one who had greater capacity than Dr. Gates. He had untiring energy, a tremendous acquaintance, a constructive imagination, and great personal charm. He was not only very active in the affairs of the University, but in many other organizations of civic interest. One of the secrets of his great capacity to serve in many things, was his ability to secure efficient help. It was considered an honor to be asked by Dr. Gates to help, for those whom he asked knew that he was in activities well worth while and they served enthusiastically without any thought of a tangible reward for themselves.

In conclusion, Mr. President, I will present to you a statement made by a close friend and associate of Charles Day and a friend of mine of many years standing—Dr. Charles Penrose:

> Charles Day, as I view his life work after an interim of twenty years, had an outstanding characteristic of a combination of originality of thinking and instant perception and analysis of an over-all situation, of whatever nature.
>
> Many times would we who were associated with him see him visit for the first time an industrial plant, grasp its overall picture, become familar in an amazingly short time with its essential operations, and then, through almost instant and original thinking, go forward to develop improvements and modifications in production methods, procedure, and approach.
>
> If you were to ask me to name Charles Day's personal attributes, I would reply: utter integrity; kindliness; inexhaustible energy; and a rare gift of inspiring his associates with enthusiasm towards a constructive objective. His was true leadership.
>
> In all things, Charles Day was thorough.
>
> In all things, Charles Day was endowed by keenest intelligence.
>
> In all things, Charles Day was a gentleman.

STATIONARY SOLUTIONS OF CERTAIN NONLINEAR DIFFERENTIAL EQUATIONS *

BY

N. MINORSKY [1]

SUMMARY

The differential equation $\ddot{x} + b\dot{x} + x + (a - cx^2)x\cos 2t + ex^3 = 0$, encountered in applications, is investigated by the perturbation theory. Equations of the first approximation are discussed in the general case as well as in a number of particular cases. In Part II the problem of the maintenance of the oscillation of a pendulum by alternating current is studied and it is shown that it is expressible in terms of this differential equation.

Part I

1. INTRODUCTORY REMARKS

One encounters in some applications a differential equation of the form

$$\ddot{x} + b\dot{x} + x + (a - cx^2)x\cos 2t + ex^3 = 0, \qquad (1.1)$$

where b, a, c, and e are small constants which we assume to be of the same order of magnitude.

One can envision (1.1) as representing a certain oscillator which, in addition to inertia ($\ddot{x}$), linear damping ($b\dot{x}$) and a restoring force (x), has also two nonlinear terms. The term ex^3 is common in nonlinear problems. As to the term $(a - cx^2)x\cos 2t$, its significance is apparent if one sets $c = e = 0$, in which case (1.1) reduces to the standard Mathieu equation by the well known transformation of the dependent variable $x = u\exp(-bt/2)$. Since a term of this nature in physical problems accounts for the initial increase of the amplitude if the parameters correspond to the unstable zone of the Mathieu equation, while the term ex^3, on the contrary, limits this increase, one sees that our oscillator possesses two kinds of nonlinearities, the *active* one and the *passive* one in a sense which is obvious from the remark just made.

The purpose of this paper is to investigate the existence of stable periodic solutions of (1.1) or, since we have in mind primarily a physical problem, of stable stationary oscillations for different values of the parameters b, a, c, and e.

We may note in passing that in nonlinear problems the conditions for the existence and stability of a solution do not always coincide. Only when both these conditions are fulfilled, does stable stationary

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oscillation actually appear. This accounts for a series of facts which may appear as paradoxical from an offhand consideration of the problem. Thus, for example an oscillation may suddenly appear (or disappear) as soon as a certain threshold of stability is crossed. In view of this we shall always investigate the conditions for the existence and for stability at the same time so as to be able to deal only with stable stationary oscillations which are the only ones which are of interest in applications.

In Part II of this paper we shall give a physical example of (1.1) relative to a pendulum whose oscillation is maintained by alternating current. This problem has been much discussed in recent years.

2. PERTURBATION METHOD

As there is no general theory for treating (1.1), we shall apply the perturbation method and will limit ourselves only to the first approximation in view of the assumed smallness of the constants b, a, c, and e. This condition is generally fulfilled in applied problems.

It is convenient to write (1.1) by setting $b = \epsilon B$, $a = \epsilon A$, $c = \epsilon C$, and $e = \epsilon E$, where ϵ is a small parameter and B, A, C, and E are finite constants of the same order of magnitude. This will enable us to study a series of particular cases by setting one or several of these constants equal to zero.

Equation 1.1 written as a system of two differential equations of the first order is

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -by - x - ax\cos 2t + cx^3\cos 2t - ex^3. \end{aligned} \tag{2.1}$$

We introduce new dependent variables defined by the relations

$$\rho = x^2 + y^2 = r^2; \quad \psi = \arctan(y/x),$$

where

$$\begin{aligned} x &= r\cos\psi; \\ y &= r\sin\psi. \end{aligned}$$

Equations 2.1 then become

$$\begin{aligned} d\rho/dt = &-b\rho + b\rho\cos 2\psi - a\rho\sin 2\psi\cos 2t \\ &+ 2c\rho^2\cos^3\psi\sin\psi\cos 2t - 2e\rho^2\cos^3\psi\sin\psi, \end{aligned} \tag{2.2}$$

$$\begin{aligned} d\psi/dt = &-1 - \tfrac{1}{2}b\sin 2\psi - a\cos^2\psi\cos 2t - e\cos^4\psi \\ &+ c\rho\cos 2t\cos^4\psi. \end{aligned} \tag{2.3}$$

We introduce an auxiliary variable $P(t) = \log\rho(t)$ and take as solution of (2.2) a series

$$P(t) = P_0(t) + \epsilon P_1(t) + \epsilon^2 P_2(t) + \cdots. \tag{2.4}$$

Replacing (2.4) into (2.2) and equating the coefficients of the various powers of ϵ, one obtains a number of differential equations which can

be integrated by a recurrence procedure. In a similar manner, as a solution of (2.3) one takes a series

$$\psi(t) = \psi_0(t) + \epsilon\psi_1(t) + \epsilon^2\psi_2(t) + \cdots. \tag{2.5}$$

The identification of terms free of ϵ yields the approximation of the zero order

$$P_0(t) = P_0; \quad \psi_0(t) = \phi_0 - t, \tag{2.6}$$

where ϕ_0 is an integration constant. In a similar manner, for the terms of the first order one gets

$$dP_1/dt = -B - B\cos 2\psi - A\sin 2\psi\cos 2t + \tfrac{1}{2}C\rho\sin 2\psi\cos 2t + \tfrac{1}{2}C\rho\sin 4\psi\cos 2t - \tfrac{1}{2}E\rho\sin 2\psi, \tag{2.7}$$

$$d\psi_1/dt = -B\sin 2\psi - \tfrac{1}{2}A\cos 2t - \tfrac{1}{2}A\cos 2\psi\cos 2t + \tfrac{3}{8}C\rho\cos 2t + \tfrac{1}{2}C\cos 2\psi\cos 2t + \tfrac{1}{8}C\rho\cos 4\psi\cos 2t - \tfrac{3}{8}E - \tfrac{1}{2}E\rho\cos 2\psi - \tfrac{1}{8}E\cos 4\psi. \tag{2.8}$$

One replaces ρ and ψ in these expressions by (2.6) and, after the integration, obtains the following expressions

$$P(t) = P_0 + \epsilon P_1(t); \quad \psi(t) = \psi_0 + \epsilon\psi_1(t). \tag{2.9}$$

3. STROBOSCOPIC IMAGE. EQUATIONS OF THE FIRST APPROXIMATION

Instead of dealing with the differential equations in the form (2.7) and (2.8) we apply the following procedure. Instead of a continuous set of points determining the integral curve in the (P, ψ) plane, we consider a sub-set of points corresponding to the instants 2π, 4π, 6π, $\cdots$. Using a pictorial language we may say that we "illuminate" the process only at these isolated instants of time and take into account only what we "see" at these instants. Since any periodic term resumes its value after 2π, it will appear as a fixed point in this "stroboscopic image" of the real phenomenon. Only those points will appear as slowly moving which correspond to the terms which *do not resume their values* after 2π, that is, the *secular terms*.

If the phenomenon lasts long enough in terms of one period 2π, we can treat in this mode of representation the *apparent* continuity of the image as a continuous process. This amounts to treating the sub-set of points just mentioned as a continuous one in this approximation.

As initial conditions we take $P_0(0) = P_0$; $\psi_0(0) = \phi_0$; $P_\nu(0) = 0$; $\psi_\nu(0) = 0$, $\nu = 1, 2 \cdots$.

From (2.7) and (2.8) it appears that the terms containing $\sin 2\psi_0\cos 2t$ or $\cos 2\psi_0\cos 2t$ are secular terms as is easy to ascertain in view of (2.6). For that reason we can drop all periodic terms in (2.7) and (2.8) and leave only the constant terms as well as the trigonometric terms just mentioned since only these terms contribute to the slow "stroboscopic" motion of the image P, ϕ. Equations 2.7 and 2.8 reduce then to the

following:

$$dP_1/dt = -B + \tfrac{1}{2}(C\rho_0 - 2A)\sin 2\phi_0 \cos^2 2t, \tag{3.1}$$

$$d\psi_1/dt = \tfrac{1}{2}(C\rho_0 - A)\cos 2\phi_0 \cos^2 2t - \tfrac{3}{8}E\rho_0. \tag{3.2}$$

After one period 2π the quantities P_1 and ψ_1 differ by the amounts

$$P_1(2\pi) = -2\pi B + \tfrac{1}{4}(C\rho_0 - 2A)\sin 2\phi_0 2\pi,$$
$$\phi_1(2\pi) = \tfrac{1}{4}(C\rho_0 - A)\cos 2\phi_0 2\pi - \tfrac{3}{8}E\rho_0 2\pi.$$

It is to be noted that these expressions are approximate with accuracy $0(\epsilon^2)$ on account of the secular terms of higher orders which can be neglected in the first approximation because the series is used only for one period, and also because the continuous time has been now replaced by the number of the step 2π in the iteration process.

The variations of $P = P_0 + \epsilon P_1$ and $\psi = \psi_0 + \epsilon\psi_1$ are

$$P(2\pi) = -2\pi\epsilon B + 2\pi\epsilon\tfrac{1}{4}(C\rho_0 - 2A)\sin 2\phi_0, \tag{3.3}$$

$$\phi(2\pi) = 2\pi\epsilon\tfrac{1}{4}(C\rho_0 - A)\cos 2\phi_0 - 2\pi\epsilon\tfrac{3}{8}E\rho_0. \tag{3.4}$$

We set $2\pi\epsilon = \Delta t$ and obtain thus two sequences of difference equations

$$\Delta P/\Delta t = -B + \tfrac{1}{4}(C\rho_0 - 2A)\sin 2\phi_0, \tag{3.5}$$

$$\Delta\phi/\Delta t = \tfrac{1}{4}(C\rho_0 - A)\cos 2\phi_0 - \tfrac{3}{8}E\rho_0, \tag{3.6}$$

connecting the subsequent intervals.

If one considers now the sub-set of points $0, 2\pi, 4\pi, \cdots$ approximately as a continuous one, one can treat Δt as dt and to pass thus from these two sequences of the difference equations to two differential equations determining the process in the long run, viz.:

$$d\rho/dt = \tfrac{1}{4}\rho[(C\rho - 2A)\sin 2\phi - 4B], \tag{3.7}$$

$$d\phi/dt = \tfrac{1}{4}[(C\rho - A)\cos 2\phi - \tfrac{3}{2}E\rho]. \tag{3.8}$$

These differential equations will be fundamental in the sequel. They are in all respects analogous to the so-called "abbreviated" equations of van der Pol. It is recalled that these latter equations are obtained by the integration between the limits of $0, 2\pi$ of a Fourier series which in the van der Pol method replaces an almost periodic series in which the amplitude and phase vary but little during one period 2π. In this procedure the same result is obtained by the elimination of the periodic terms in the "stroboscopic image" of (2.7) and (2.8).

4. STATIONARY STATE AND ITS STABILITY

The singular point of (3.7) and (3.8) gives the stationary amplitude ρ_0 and phase $2\phi_0$. The latter is given by the expressions

$$\sin 2\phi_0 = 4B/(C\rho_0 - 2A); \quad \cos 2\phi_0 = 3E\rho_0/2(C\rho_0 - A). \tag{4.1}$$

The stationary amplitude ρ_0 is obtained from the expression $\sin^2 2\phi_0 + \cos^2 2\phi_0 = 1$ in which (4.1) is used. This results in a quartic equation

$$4(C\rho_0 - A)^2(C\rho_0 - 2A)^2 - 64B^2(C\rho_0 - A)^2 - 9E^2\rho_0^2(C\rho_0 - 2A)^2 = 0. \quad (4.2)$$

Clearly, only the real and positive roots of (4.2) are of interest here.

The stability of the stationary state is determined by the variational equations obtained by replacing ρ and ϕ in (3.7) and (3.8) by $\rho_0 + \delta\rho$ and $\phi_0 + \delta\phi$ which leads to the following *characteristic equation:*

$$S^2 + BS - \tfrac{1}{4}[(C\rho_0 - A)^2 \sin^2 2\phi_0 - 2B(C\rho_0 - A)\sin 2\phi_0 + \tfrac{1}{2}C\rho_0(C\rho_0 - 2A)\cos^2 2\phi_0 - \tfrac{3}{4}E\rho_0(C\rho_0 - 2A)\cos 2\phi_0] = 0. \quad (4.3)$$

The problem of stability reduces to the determination of the sign of the square bracket Q in (4.3) into which the real and positive root of (4.2) has been replaced for ρ_0. If $Q < 0$, the singular point of (3.7), (3.8) is a stable focal, or a stable nodal point so that the stable stationary oscillation exists. If $Q > 0$, it is a saddle point and the stable stationary oscillation is absent. Algebraically, the problem is too complicated to be of any practical interest but if the numerical values of the parameters B, A, C, and E are given, it does not present any particular difficulty. We will return to this question in Part II.

5. PARAMETER SPACE

We shall investigate now a series of particular cases of (1.1) by setting one or several parameters equal to zero. This gives rise to a number of differential equations which are of interest in applications and which we shall be able to discuss more fully than the general case.

For the purpose of a classification of these particular cases it is convenient to consider certain regions in the four-dimensional space of parameters B, A, C, and E, ruling out $B < 0$ which corresponds to the unstable singular points, as is seen from (4.3). Moreover, physically this case is not of any interest either, B being always positive in applications.

We will consider two major groups of particular cases: one for $C > 0$ and the other for $C < 0$. Each group contains 14 differential equations. To this one must add the 29th case when $C = 0$.

Each group contains the following cases:

(1), (2) $B = 0, A > 0, E \gtrless 0$; (3), (4) $B = 0, A < 0, E \gtrless 0$;
(5), (6) $A = 0, B > 0, E \gtrless 0$; (7), (8) $E = 0, B > 0, A \gtrless 0$;
(9), (10) $E = B = 0, A \gtrless 0$; (11) $A = E = 0, B > 0$;
(12), (13) $A = B = 0, E \gtrless 0$; (14) $A = B = E = 0$. (5.1)

In this table, the particular cases are arranged by pairs; the first differential equation in each pair corresponds to the sign + of the parameter subject to double sign and the second, to the sign −. In most cases we shall investigate only the first differential equation in each pair of cases since the second generally follows the same argument although the results are, as a rule, entirely different. In some cases the second differential equation presents entirely different qualitative aspects of its solutions and for that reason we shall occasionally analyse the second as well.

6. PARTICULAR CASES FOR $C > 0$

In each particular case we shall indicate the form of the corresponding differential equation which will help to identify the physical problems involved. In each case we will investigate the conditions for the existence of the stationary amplitude and its stability. When both these conditions are fulfilled, we will be able to assert that *the stable stationary oscillation exists*. This restricts the investigation only to cases which are of a physical interest. The procedure consists of: (*a*) determining ρ_0 from (4.2), (*b*) determining $\sin 2\phi_0$, $\cos 2\phi_0$ from (4.1), and (*c*) establishing the conditions under which Q is negative. In step (*a*) there arises frequently a condition expressing the fact that ρ_0 is positive; we will refer to it as *necessary condition*. In some cases there appear alternatives arising from a double sign of certain quantities. In such cases the condition of stability permits making a proper choice between the two alternatives.

Cases (1) and (2): $\ddot{x} + x + (a - cx^2)x \cos 2t + ex^3 = 0$ (6.1)

This is a differential equation of a nonlinear conservative ($B = 0$) system possessing nonlinearities both of active and passive types. In case (1) we have, from (4.2),

$$\rho_0 = A/(C \pm 3E/2). \tag{6.2}$$

This shows that there are two stationary amplitudes corresponding to two signs in the denominator. Call the one corresponding to the sign +, the first amplitude (or oscillation) and corresponding to the sign −, the second. The first oscillation exists for any value of $A > 0$ and $E > 0$ and the second only for such values which render the denominator positive. This gives the necessary condition for the second oscillation

$$E < 2C/3. \tag{6.3}$$

From (4.1) we have $\sin 2\phi_0 = 4B/(C\rho_0 - 2A) = 0$, hence $\cos 2\phi_0 = \pm 1$. From (4.3)

$$Q = \tfrac{1}{2}\rho_0(C\rho_0 - 2A)(C \cos 2\phi_0 - 3E/2) \cos 2\phi_0. \tag{6.4}$$

For stability Q must be negative. We have an alternative:

$$\cos 2\phi_0 = 1, \quad \cos 2\phi_0 = -1.$$

For $\cos 2\phi_0 = +1$ we have $Q = \frac{1}{2}\rho_0(C\rho_0 - 2A)(C - 3E/2)$. The brackets must be, therefore, of opposite signs. Since the second bracket is positive, by (6.3), the condition of stability reduces to

$$\rho_0 < 2A/C. \tag{6.5}$$

Comparing with (6.2), it is seen that (6.5) is fulfilled for the first oscillation if $A/(C + 3E/2) < 2A/C$. This condition is obviously fulfilled for any value of $A > 0$ and $C > 0$.

As to the second oscillation, (6.5) corresponds to the inequality $A/(C - 3E/2) < 2A/C$ which reduces clearly to the condition

$$E < C/3. \tag{6.6}$$

Comparing with (6.3), it is seen that (6.6) gives the necessary and sufficient conditions for the existence of the second stable stationary oscillation.

Summing up, if $0 < E < C/3$, both oscillations exist with amplitudes given by (6.2) but, since these oscillations have the same frequency and phase, what one observes *physically* is the resultant oscillation equal to their algebraic sum. On crossing the critical threshold (6.6), the second oscillation disappears. The physical indication of this fact will be a certain "jump" in the amplitude of the resulting oscillation when this critical threshold is crossed.

For $\cos 2\phi_0 = -1$, $Q = \frac{1}{2}\rho_0(C\rho_0 - 2A)(C + 3E/2)$ which gives the condition of stability $\rho_0 < 2A/C$ which is the same condition (6.5). We conclude, therefore that, as far as the stationary phase is concerned, the stable stationary oscillation may exist with any phase $\phi_0 = 0$, $\phi_0 = \pi/2$ etc.

As regards the case (2), one has to replace E by $-|E|$ the remaining argument being the same; the results are obviously entirely different.

Cases (3) and (4): $\ddot{x} + x = (a + cx^2)x\cos 2t + ex^3 = 0$ (6.7)

From a physical point of view, this case (3) represents a system similar to that investigated in case (1) but with a different relative phase of the active components $ax\cos 2t$ and $cx^3\cos 2t$. From (4.2) we get

$$\rho_0 = |A|/(3E/2 - C),$$

with the necessary condition $E > 2C/3$. From (4.1) $\sin 2\phi_0 = 0$, $\cos 2\phi_0 = \pm 1$ and from (4.3)

$$Q = (\rho_0/2)(C\rho_0 + 2|A|)(C\cos 2\phi_0 - 3E/2)\cos 2\phi_0.$$

For $\cos 2\phi_0 = +1$ the condition of stability requires $C - 3E/2 < 0$ which is precisely the necessary condition which is thus also a sufficient one. Hence, a stable stationary oscillation exists if $E > 2C/3$.

The case (4) is analyzed in a similar manner.

Cases (5) and (6): $\ddot{x} + b\dot{x} + (1 - cx^2 \cos 2t)x + ex^3 = 0$ (6.8)

From (4.2) one gets

$$\rho_0 = 8B/\sqrt{4C^2 - 9E^2}, \tag{6.9}$$

with the necessary condition $E < 2C/3$. Moreover, $\sin 2\phi_0 = 4B/C\rho_0$; $\cos 2\phi_0 = 3E/2C$. If one replaces these values into the expression for Q one finds

$$Q = 8B^2 > 0, \tag{6.10}$$

which shows that there is no stable stationary oscillation.

Cases (7) and (8): $\ddot{x} + b\dot{x} + [1 + (a - cx^2) \cos 2t]x = 0$ (6.11)

We study first the case (7). The stationary amplitudes in this case are

$$\rho_0 = (2A + 4B)/C \quad \text{and} \quad \rho_0 = (2A - 4B)/C. \tag{6.12}$$

The necessary condition for the second amplitude is

$$A > 2B. \tag{6.13}$$

From (4.1) one has $(C\rho_0 - A)\cos 2\phi_0 = 0$. Since $(C\rho_0 - A) \neq 0$, $\cos 2\phi_0 = 0$ and, therefore $\sin 2\phi_0 = \pm 1$. For $\sin 2\phi_0 = +1$, one has the first value (6.12), and for $\sin 2\phi_0 = -1$, one has the second value. One has thus an alternative (*a*) or (*b*):

(*a*) For $\sin 2\phi_0 = +1$, $Q = (C\rho_0 - A)(C\rho_0 - A - 2B)$. For stability the bracket must have opposite signs and, since $B > 0$, the first bracket must be positive and the second, negative. The first inequality is obviously fulfilled in view of the first equation (6.12). Hence, the condition of stability depends on $(C\rho_0 - A - 2B) < 0$, that is, $\rho_0 < (A + 2B)/C$. As this condition is contradictory to the first equation (6.12), the stability condition is not fulfilled and, therefore, no stable stationary oscillation exists in this case.

(*b*) For $\sin 2\phi_0 = -1$, $Q = (C\rho_0 - A)(C\rho_0 - A + 2B)$. For stability the brackets must be of opposite signs. This gives $(C\rho_0 - A) = A - 4B$ and $(C\rho_0 - A + 2B) = A - 2B$. Since $A - 2B > 0$ by (6.13), one must have $A - 4B < 0$ which gives

$$2B < A < 4B. \tag{6.14}$$

If A is in this interval, a stable stationary oscillation is possible with $\rho_0 = (2A - 4B)/C$ and the phase angle $\phi_0 = 3\pi/4$.

In the case (8) there is only one value $\rho_0 = 4B - 2|A|/C$ with the necessary condition $2B > |A|$. From (4.1) one finds $\cos 2\phi_0 = 0$, $\sin 2\phi_0 = +1$, the sign minus being ruled out by the fact that all terms in the expression for $\sin 2\phi_0$ are positive.

In this case, $Q = (C\rho_0 + |A|)(C\rho_0 + |A| - 2B)$ which shows that the condition of stability can be fulfilled only if $C\rho_0 + |A| < 2B$. This is, however, impossible in view of the necessary condition. For that reason, no stable stationary oscillation exists in this case.

Cases (9) and (10): $\ddot{x} + x + (a - cx^2)x \cos 2t$ (6.15)

This differential equation can be regarded as a nonlinear Mathieu equation if one writes it in the form

$$\ddot{x} + (1 + (a - cx^2)\cos 2t)x = 0.$$

If x is small (that is, cx^2 negligible) this is a Mathieu equation with the parametric point $(1, a)$ situated in the unstable zone of the parameter plane (1) which shows that the amplitude increases from rest. This was the reason why we have designated the term $(a - cx^2)x \cos 2t$ as a nonlinear *active* term inasmuch as in a physical problem it accounts for the energy input into the oscillator.

In the case (9) we have from (4.2): $(C\rho_0 - A)^2(C\rho_0 - 2A)^2 = 0$, whence the alternative: either $\rho_0 = A/C$ or $\rho_0 = 2A/C$. From (4.1) one gets

$$(C\rho_0 - 2A)\sin 2\phi_0 = 0; \quad (C\rho_0 - A)\cos 2\phi_0 = 0.$$

The alternative in question is, therefore, either

$$(C\rho_0 - 2A) = 0, \quad \cos 2\phi_0 = 0, \quad \sin 2\phi = \pm 1;$$

or

$$(C\rho_0 - A) = 0, \quad \sin 2\phi_0 = 0, \quad \cos 2\phi_0 = \pm 1.$$

For the first alternative one finds $Q = (C\rho_0 - A)^2$, which rules out the existence of a stable stationary oscillation. For the second alternative $Q = \frac{1}{2}C\rho_0(C\rho_0 - 2A) < 0$, which means that in this case a stable stationary oscillation exists with $\rho_0 = A/C$ and with either one of the phases $\cos 2\phi_0 = +1$ or $\cos 2\phi_0 = -1$.

One finds that in the case (10) the stable stationary oscillation does not exist.

Case (11): $\ddot{x} + b\dot{x} + x - cx^3 \cos 2t = 0$ (6.16)

In this case, $\rho_0 = 4B/C$; $\sin 2\phi_0 = +1$; $Q = C\rho_0 2B > 0$ which rules out the existence of a stable stationary oscillation.

Cases (12) and (13): $\ddot{x} + x - cx^3 \cos 2t + ex^3 = 0$ (6.17)

From (4.2) one finds that $\rho_0 = 0$ unless $E = 2C/3$. This means that the case (12) is a singular one inasmuch as it requires an *exact* relation

between the parameters E and C. Moreover, ρ_0 is not determined by either (4.1) or (4.2). It is, therefore, of no interest.

Case (14): $\ddot{x} + x - cx^3 \cos 2t = 0$ (6.18)

This is also a special case since the differential equations (3.7) and (3.8) indicate the absence of a stationary state. One notices this directly from (2.7) and (2.8) which show the presence of only one secular term which cannot, therefore, be eliminated so that no stable stationary oscillation is possible here either.

7. PARTICULAR CASES FOR $C < 0$

We shall investigate some of these cases and compare them with the corresponding cases for $C > 0$. As a rule, a relatively simple case (for example, no alternatives) for $C > 0$ becomes complicated for $C < 0$ and vice versa. This is due to the fact that the condition of stability usually appears in the form of a product of two brackets. If these can have two different signs, one has an alternative. The change of sign of C generally removes one sign from one bracket so that the matter is settled by the condition to be imposed on the sign of the other bracket.

We will investigate first the case $C = 0$ which is the 29th case in this classification.

Case $C = 0$: $\ddot{x} + b\dot{x} + (1 - a \cos 2t)x + ex^3 = 0$ (7.1)

This differential equation has been studied in an earlier paper (2)[2] but, for the sake of completeness we will review it here. It represents a dissipative ($b\dot{x}$) oscillator with a linear Mathieu term $(1 - a \cos 2t)x$ and with a nonlinear passive term ex^3. If $e = 0$, it becomes a Mathieu equation after a change of the dependent variable defined by the relation $x = u \exp(-bt/2)$.

From (4.2) we have $\rho_0 = (2/3E)\sqrt{A^2 - 4B^2}$ with the necessary condition $A > 2B$. From (4.1), $\sin 2\phi_0 = -(2B/A)$; $\cos 2\phi_0 = -(3E\rho_0/2A)$. From (4.3), $Q = -9E^2\rho_0^2/2$. Hence, the necessary condition is also a sufficient one here provided $E > 0$. No stable stationary oscillation is possible if $E = 0$ which is obvious since in such a case (7.1) becomes an ordinary Mathieu equation in the unstable zone. For $E < 0$, ρ_0 does not exist.

Case (1a): $\ddot{x} + x + (a + |c|x^2)x \cos 2t + ex^3 = 0$ (7.2)

In this case, $\rho_0 = 2A/(3E - 2|C|)$, the second sign before E having been eliminated by the requirement $\rho_0 > 0$. The necessary condition is $E > 2|C|/3$. We have also $\sin 2\phi_0 = 0$; $\cos 2\phi_0 = \pm 1$; $Q = (1/2)\rho_0(|C|\rho_0 + 2A)(|C| \cos 2\phi_0 + 3E/2) \cos 2\phi$. Hence, for $\cos 2\phi_0 = +1$ there is no stability; for $\cos 2\phi_0 = -1$ stability exists

[2] The boldface numbers in parentheses refer to the references appended to this paper.

provided $E > 2|C|/3$. Here again the necessary condition is also sufficient. It is noted that this case is simpler than the case (1) in view of the absence of the alternatives.

Case (3a): $\ddot{x} + x - (a - cx^2)x\cos 2t + ex^3 = 0$ (7.3)

This case, on the contrary, is more complicated than the relatively simple case 3. We have

$$\sin 2\phi_0 = 0; \quad \cos 2\phi_0 = \pm 1 = 3E\rho_0/2(|A| - |C|\rho_0).$$

For $\cos 2\phi_0 = +1$, $\rho_0 = 2|A|/(2|C| + 3E)$; for $\cos 2\phi_0 = -1$, $\rho_0 = 2|A|/(2|C| - 3E)$, with the necessary condition $E < 2|C|/3$; $Q = -(1/2)\rho_0(2|A| - |C|\rho_0)(|C| - 3E\cos 2\phi_0)$.

For $\cos 2\phi_0 = +1$, $Q = -(1/2)\rho_0(2|A| - |C|\rho_0)(|C| - 3E)$, whence the alternative: either

$$2|A| - |C|\rho_0 > 0, \quad E < |C|/3$$

or

$$2|A| - |C|\rho_0 < 0, \quad E > |C|/3.$$

The first alternative gives $\rho_0 < 2|A|/|C|$; this inequality is satisfied for ρ_0 corresponding to $\cos 2\phi_0 = +1$ for any values of $|A|$, $|C|$, and E so that the necessary condition $E < |C|/3$ is also a sufficient condition. The second alternative requires $\rho_0 > 2|A|/|C|$ and is to be rejected since $\rho_0 < 2|A|/|C|$ for the above value of ρ_0.

For $\cos 2\phi_0 = -1$, $Q = -(1/2)\rho_0(2|A| - |C|\rho_0)(|C| + 3E)$ so that the condition of stability requires $\rho_0 < 2|A|/|C|$. With ρ_0 corresponding to $\cos 2\phi_0 = -1$ this requirement is possible if $|C| - 3E > 0$, that is $E < |C|/3$, which is thus the necessary and sufficient condition for the existence of the stable stationary oscillation.

Summing up, a stable stationary condition exists for $E < |C|/3$ with either one of the two phases $\cos 2\phi_0 = +1$ and -1 and does not exist if $E > |C|/3$.

Cases 7*a*, 8*a*, 9*a*, and 10*a* follow the same argument. The complicated cases for $C > 0$, become simple for $C < 0$ and vice versa. Case (11*a*) gives again a negative result but the reason for that is different; in the case (11) no stable stationary oscillation exists because the condition for stability is not fulfilled. Here it does not exist because the stationary amplitude does not exist. Cases (12*a*) and (13*a*) remain singular for $C < 0$ and (14*a*) does not have any stationary solution as before.

8. STABILITY OF THE STATE OF REST ($\rho = 0$). SELF-EXCITATION

We shall supplement the preceding study by the investigation of stability of the state of rest. It is obvious that, if an oscillatory system represented by (1.1) is to be able to start from rest (or to be "self-excited"), the point of rest $x = 0$ must be unstable.

In the neighborhood of $\rho = 0$, the system (3.7), (3.8) can be written as

$$d\rho/dt = -\tfrac{1}{2}\rho(A \sin 2\phi + 2B), \tag{8.1}$$

$$d\phi/dt = -\tfrac{1}{4}(A \cos 2\phi + 3E\rho/2). \tag{8.2}$$

The state of rest $\rho = 0$ is a singular point of this system provided $\cos 2\phi_0 = 0$, that is, $2\phi_0 = \pi/2$ or $3\pi/2$.

The variational equations of (8.1) and (8.2) lead to the following characteristic equation

$$S^2 + BS - (1/4)\, A \sin 2\phi_0(A \sin 2\phi_0 + 2B) = 0. \tag{8.3}$$

For $2\phi_0 = \pi/2$, (8.3) yields:

$$S^2 + BS - (1/4)\, A(A + 2B) = 0, \tag{8.4}$$

and for $2\phi_0 = 3\pi/2$

$$S^2 + BS + (1/4)(2B - A) = 0. \tag{8.5}$$

For $A > 0$, (8.4) shows that the system (8.1), (8.2) has a saddle point, whereas (8.5) indicates that a saddle point exists if $2B < A$.

For $A < 0$, the above conditions are reversed, viz.: (8.4) shows that a saddle point exists if $2B < A$ and in the case (8.5) there is always a saddle point.

All depends, therefore, on whether $2\phi_0 = \pi/2$, or $3\pi/2$. The choice between these two values of $2\phi_0$ is dictated by the condition of stability of the phase. From (8.2) where we set $\rho = 0$ we get

$$d\phi/dt = -(1/4)\, A \cos 2\phi, \tag{8.6}$$

and it is seen that in the neighborhood of $\cos 2\phi_0 = 0$ the variational equation is

$$d\delta\phi/dt = (1/2)\, A \sin 2\phi\delta\phi. \tag{8.7}$$

This shows that for $A > 0$, the phase $2\phi_0 = \pi/2$ is unstable and $2\phi_0 = 3\pi/2$ is stable. For $A < 0$, on the contrary, $\pi/2$ is stable and $3\pi/2$ unstable.

Hence in each case the phase will approach its stable value and the condition of self-excitation is

$$2B < |A|. \tag{8.8}$$

It is noted that this condition involves only the coefficients of the linear terms as is to be expected on the basis of the Liapounoff theorem.

Moreover, since B is a *passive* parameter (section 1), it is obvious that A is an *active* one corresponding to the term through which the energy is introduced into the system so that (8.8) acquires an almost intuitive significance.

9. SINGULAR POINTS AND INTEGRAL CURVES OF THE (ρ, ϕ) PLANE

The concept of singular points as positions of equilibrium was used in two different connections in the preceding study, viz.: equilibrium of a stationary motion (sections 3 to 7) and the equilibrium of the state of rest (section 8).

In the first case our aim was to determine the condition of stability, and in the second, that of instability (that is, self-excitation).

In this manner, the behavior of a system is specified for the beginning of the oscillatory process and for its ultimate stationary state, but the intermediate region between the two cannot be determined on the basis of the differential equation of the first approximation. In applications, this question is of no special interest.

It is useful to specify certain properties of singular points and integral curves in the (ρ, ϕ) plane with which we have conducted this study.

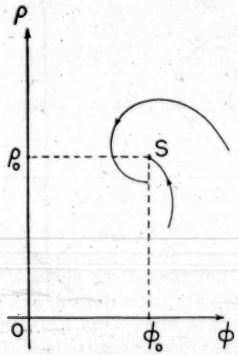

FIG. 1.

The first part of the problem, the stationary state, is represented in this plane by a point S of coordinates (ρ_0, ϕ_0) and its stability is characterized by the behavior of integral curves in its neighborhood. In this study it is convenient to consider ρ and ϕ as Cartesian coordinates, at least in a certain neighborhood of S (Fig. 1).

If $Q < 0$ in (4.3), the stationary state is stable and the integral curves are either spirals (Fig. 1) approaching S (case of a focal point) or distorted parabolic curves approaching S from some definite directions (nodal point). In both cases the stability exists since $B > 0$ in (4.3). If $Q > 0$, the system (3.7), (3.8) has a saddle point so that no stable stationary oscillation is possible.

The second part of the problem, the stability of the state of rest, is characterized, as we saw, by the existence of a saddle point at the origin ($\rho = 0$) if the condition (8.8) is fulfilled. If not, the singular point is

stable and the oscillatory process cannot start from rest. For the representation of integral curves in the large, it is convenient to consider ρ and ϕ as polar coordinates. In this case we have a configuration shown in Figs. 2 and 3. An integral curve originates from the saddle point at the origin and either winds itself onto the point S of coordinates ρ_0 and ϕ_0 in a spiral manner[3] (Fig. 2, focal point) or approaches S from some definite direction (Fig. 3, nodal point). The intermediate part of the curve (broken line) is not determined in this case as was just mentioned. If $B = 0$ and $Q < 0$ in (4.3), the stability of the stationary motion is either of the *center* type or a special focal point characterized by the relation $S_1 + S_2 = 0$ between the roots of (4.3). It is impossible to discriminate between the two on the basis of the variational equations of the first order, and this leads to a far more difficult problem investigated by Poincaré and Bendixson.[4] Without going into this

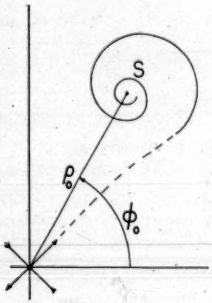

FIG. 2

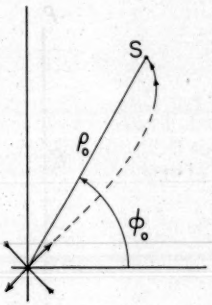

FIG. 3.

matter here, it is sufficient to mention that the integral curves in the neighborhood of S are closed curves containing S in their interior. This means that the stationary state in this case is slightly "floating" with parasitic amplitude and phase modulations under the effect of accidental disturbances. Since in any physical problem $B \neq 0$, this case is of a purely theoretical interest.

Part II

10. MAINTENANCE OF MECHANICAL OSCILLATIONS BY ALTERNATING CURRENT

As an illustration of the differential equation (1.1) as encountered in physics we shall investigate a somewhat curious phenomenon which has raised considerable discussion in recent years.

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[3] A. A. Andronow and C. E. Chaikin, "Theory of Oscillations," Princeton University Press, 1949, p. 339, Fig. 310, 9.

[4] Cf. L. Bieberbach, "Differentialgleichungen," Springer, 1930, Ch. IV.

If a pendulum provided with a piece of a ferromagnetic material is placed in the neighborhood of an induction coil L carrying an alternating current in the manner shown in Fig. 4, it is frequently observed that it starts oscillating and reaches a stationary state of oscillation.

A series of early attempts to explain this effect did not throw much light on its physical nature until, finally, Y. Rocard formulated, on physical grounds, a differential equation of this phenomenon which seemed to be plausible. Strangely enough, in his treatment of this subject (3), Rocard did not make use of his own differential equations, but limited himself to a rather qualitative discussion.

Although, as far as the author knows, there exists no method for integrating his differential equations, it can be shown that they can be reduced to (1.1) at least in the first approximation, which seems sufficient on physical grounds.

FIG. 4.

Besides this, in his analysis of the subject, Rocard limited himself to the case of asymmetrical inductance. In reality, most of the experiments made so far relate to a symmetrical inductance and, as will be shown, this case is of much greater interest than the asymmetrical case.

In this Part II, we propose to investigate this matter.

11. DIFFERENTIAL EQUATIONS OF ROCARD

These differential equations are

$$d(L(\theta)i)/dt + zi = E\sin\omega t, \tag{11.1}$$

$$J\ddot{\theta} + D\dot{\theta} + C\theta = d/d\theta(\tfrac{1}{2}Li^2), \tag{11.2}$$

where z, E, J, D, and C are positive constants and $L(\theta)$, the inductance, is a certain function of θ, the angle of the pendulum.

The physical significance of these differential equations is sufficiently obvious: the oscillation of the pendulum modifies the inductance $L(\theta)$ of the electric circuit specified by (11.1) and the pondero-motive reaction arising from this modulation of $L(\theta)$ and represented by the right hand term in (11.2) modifies the natural motion of the pendulum specified by the left hand term in (11.2).

In spite of the physical simplicity of the problem, the system of differential equations (11.1), (11.2) cannot be integrated exactly by any known method and it is necessary, therefore, to adopt a somewhat indirect procedure which consists essentially of the following:

We assume first the existence of a periodic solution of (11.2) and determine the solution of (11.1) under this assumption. This solution having been substituted into (11.2), reduces it to the form (1.1) whose properties have been investigated in Part I. In this manner, the initial assumption of periodicity of solutions will be justified by an a posteriori argument.

12. ELECTRICAL DIFFERENTIAL EQUATION (11.1)

We assume that $L(\theta)$ is a symmetrical function of the form

$$L(\theta) = L_0 - \alpha_2\theta^2 - \alpha_4\theta^4. \tag{12.1}$$

The choice of this function is taken on physical grounds in view of the fact that $L(\theta)$ is obviously maximum for $\theta = 0$. Moreover, $L(\theta)$ remains the same if θ is replaced by $-\theta$ for the symmetrical case which we assume.

It is also obvious that $\alpha_2/L_0 = b_2 \ll 1$ and $\alpha_4/L_0 = b_4 \ll 1$ in view of the experimental fact that normally the oscillation of the pendulum modifies somewhat the leakage flux around the inductance coil and since this flux is generally a small fraction of the total flux linking with the coil, the above assumption is sufficiently plausible. Moreover, the amplitude of oscillation θ_0 is generally of the order 1/5 (in radians) at the most, so that the terms like $b_2\theta^2$ and $b_4\theta^4$ which we shall encounter later are small terms compared to unity.

Using (12.1) and introducing certain simplifications arising from the order of magnitude of the various quantities just mentioned, (11.1) can be written in the form

$$di/dt - [2b_2\theta\dot{\theta} + 4b_4\theta^3\dot{\theta} - \beta](1 + b_2\theta^2)i = m(1 + b_2\theta^2)\sin\omega t, \tag{12.2}$$

where $\beta = z/L_0$; $m = E/L_0$ and the factor $(1 + b_2\theta^2)$ arises from the division by L, where we treat $b_2\theta^2$ as small compared to 1. One can replace this factor by 1 on the left side of (12.2) since it multiplies a small quantity in the square bracket, while on the right side we keep it as it is, since m is not small.

We assume now $\theta = \theta_0 \cos\Omega t$ where Ω is the angular frequency of the pendulum which is yet undetermined. We have thus a linear differential equation of the first order with a periodic coefficient. Replacing θ, θ^2, θ^3, and $\dot{\theta}$ by their values, (12.2) becomes

$$di/dt - (\beta + a_2\sin 2\Omega t + a_4\sin 4\Omega t)i = M\sin\omega t + \tfrac{1}{2}N\sin(\omega + \Omega)t + \tfrac{1}{2}N\sin(\omega - \Omega)t, \tag{12.3}$$

where $a_2 = (b_2\theta_0^2 + b_4\theta_0^4)\Omega$; $a_4 = \frac{1}{2}b_2\theta_0^4\Omega$; $M = m(1 + b_2\theta_0^2/2)$; and $N = mb_2\theta_0^2/2$. This differential equation is of the form $di/dt + f(t)i$

$= g(t)$ whose general solution is

$$i = \exp(-F(t)) \int g(t) \exp(F(t))\, dt,$$

where $F(t) = \int f(t)\, dt = \beta t - (a_2/2\Omega) \cos 2\Omega t - (a_4/4\Omega) \cos 4\Omega t$ and $\exp(\pm F(t)) \cong e^{\pm \beta t}(1 \pm (a_2/2\Omega) \cos 2\Omega t \pm (a_4/4\Omega) \cos 4\Omega t + \cdots)$. In this expression the exponential of the trigonometric terms has been replaced by the first two terms of its Taylor's expansion since a_2 and a_4 are small.

Carrying out this simple but lengthy calculation one obtains the solution of (12.3) in the form

$$i = i_0 \cos\omega t + i_2 \cos(\omega + 2\Omega)t + i_2' \cos(\omega - 2\Omega)t + i_4 \cos(\omega + 4\Omega)t + \cdots. \quad (12.4)$$

In this series $i_0, i_2, i_2' \cdots$ can be calculated in terms of the constants and parameters b_2, b_4, a_2, a_4, M, and N, but for our purpose this is not necessary. It is sufficient to mention only that i_0 is the quantity of the zero order of smallness; i_2 and i_2' are of the first order; i_4 and i_4' are of the second order, etc., so that the subsequent terms decrease rapidly in view of the assumed smallness of the parameters.

13. COUPLING TERMS

We shall calculate now the right hand term in (11.2). The value of i^2 is

$$i^2 = \tfrac{1}{2}(i_0^2 + i_2^2 + i_2'^2 + \cdots) + \tfrac{1}{2}(i_0^2 \cos 2\omega t + i_2^2 \cos 2(\omega + 2\Omega)t + i_2'^2 \cos 2(\omega - 2\Omega)t + \cdots) + (i_0 i_2 (\cos 2\Omega t + \cos 2(\omega + \Omega)t) + i_0 i_2' (\cos 2\Omega t + \cos 2(\omega - \Omega)t + \cdots). \quad (13.1)$$

The first two groups of terms arise from the squares of the terms in (12.4) and the last group represents the double cross-products. It is precisely this last group which is of interest here inasmuch as these cross-products yield the terms like $i_0 i_2 \cos 2\Omega t$, $i_0 i_2' \cos 2\Omega t$, $i_0 i_4 \cos 4\Omega t$, etc., *which do not contain the frequency ω of the alternating current but contain only the frequency Ω of the pendulum.* We may call these terms the *synchronous* (with the pendulum) *terms*.

The major point of the problem is to show that these synchronous terms are able to maintain the oscillation of the pendulum.

Setting $q_0 = (1/2)(i_0^2 + i_2^2 + \cdots)$, $q_2 = i_0(i_2 + i_2')$, $q_4 = i_0(i_4 + i_4')$, etc., one gets a more compact expression for (13.1):

$$i^2 = q_0 + q_2 \cos 2\Omega t + q_4 \cos 4\Omega t + \cdots, \quad (13.2)$$

in which all terms containing ω are dropped since they do not produce any action on the pendulum.

We are able now to calculate the coupling term

$$d/d\theta(\tfrac{1}{2}Li^2) = \tfrac{1}{2}i^2(dL/d\theta) + \tfrac{1}{2}L(di^2/d\theta).$$

The second term on the right hand side of this expression is zero since i^2 does not depend on θ explicitly (in fact, it depends on θ only through the differential equation). With this simplification we get:

$$d/d\theta(\tfrac{1}{2}Li^2) = -L_0[q_0b_2\theta + 2q_0b_4\theta^3 + (q_2b_2 + 2q_4b_4\theta^2)\theta\cos2\Omega t + (q_4b_2 + 2q_4b_4\theta^2)\theta\cos4\Omega t + \cdots]. \quad (13.3)$$

14. MECHANICAL DIFFERENTIAL EQUATION (11.2)

Setting (13.3) in (11.2) and re-grouping the terms, one gets (after division by J)

$$\ddot{\theta} + b'\dot{\theta} + \Omega^2\theta + e'\theta^3 + (a' + c'\theta^2)\theta\cos2\Omega t = 0, \quad (14.1)$$

where $b' = D/J$; $\Omega^2 = (C + \alpha_2q_0)/J$; $e' = 2\alpha_4q_0/J$; $a' = \alpha_2q_2/J$; $c' = 2\alpha_4q_2/J$.

In (14.1) we conserve only the parametric term with $\theta\cos2\Omega t$ since only this term accounts for the parametric action of the first order (2). The angular frequency of the pendulum Ω which remained so far undetermined is given now by the expression $\Omega = \sqrt{(C + \alpha_2q_0)/J}$, differing from the mechanical frequency $\Omega_m = \sqrt{C/J}$ in view of the electrical term α_2q_0.

Introducing as a new independent variable $\tau = \Omega t$, (14.1) becomes

$$\ddot{\theta} + b\dot{\theta} + \theta + e\theta^3 + (a + c\theta^2)\theta\cos2\tau = 0, \quad (14.2)$$

where b, e, a, and c are b', e', a', and c' divided by Ω^2. This differential equation is exactly (1.1) with $c < 0$ so that, instead of c, one should write $|c|$. On the other hand, $c = 2\alpha_4q_2/J\Omega^2$ and, if $c < 0$, α_4 must be negative since other factors are positive. Since α_4 is negative, E is also negative. Setting the signs of the negative coefficients explicitly, (14.2) becomes

$$\ddot{\theta} + b\dot{\theta} + \theta + (a + |c|\theta^2)\theta\cos2t - |e|\theta^3 = 0. \quad (14.3)$$

With $\alpha_4 < 0$, (12.1) is

$$L(\theta) = L_0 - \alpha_2\theta^2 + |\alpha_4|\theta^4. \quad (14.4)$$

It is easy to show that (14.4) is plausible on physical grounds. In fact, for $|\theta|$ small, the inductance decreases relatively rapidly when $|\theta|$ increases because the piece of iron, being near the coil, modifies the principal part of the flux linking with the coil. For larger $|\theta|$ the iron is in the region of the stray flux and, for that reason, for the same increase of $|\theta|$, $L(\theta)$ decreases less rapidly. It is obvious that (14.4) expresses precisely this fact by the positive term $\alpha_4\theta^4$ which is negligible initially but which, for larger $|\theta|$ begins to *reduce* the decrease of $L(\theta)$ caused by the term $-\alpha_2\theta^2$.

15. SELF-EXCITATION AND MAINTENANCE OF OSCILLATION

It remains to show that (14.3), which is of the form (1.1) with c and e (and, therefore, C and E) negative, possesses a stable stationary solution. We assume $A > 0$; as to B, it is always positive.

Taking into account the signs, (4.1) yields

$$\sin 2\phi_0 = -4B/(|C|\rho_0 + 2A) < 0;$$
$$\cos 2\phi_0 = 3|E|\rho_0/2(|C|\rho_0 + A) > 0. \quad (15.1)$$

After making explicit the signs of the various quantities entering into the expression for Q in (4.3), we obtain

$$Q = -|\sin 2\phi_0|(|C|\rho_0 + A)[2B - |\sin 2\phi_0|(|C|\rho_0 + A)]$$
$$+ \tfrac{1}{2}|C|\rho_0(|C|\rho_0 + 2A)x \cos 2\phi_0[\cos 2\phi_0 - 3|E|/2|C|]. \quad (15.2)$$

The condition of stability is $Q < 0$ as before. In order to simplify the notation we shall write from now on $\sin 2\phi_0$, C and E instead of $|\sin 2\phi_0|\,|C|$ and $|E|$, having in mind that these quantities are *positive*, the proper signs having been taken into account in (15.2).

In view of the complexity of the general case with which we are dealing now, we shall establish only a *sufficient* condition for stability. For this purpose, we shall require that the first square bracket in (15.2) be positive and the second negative. This will give a sufficient condition for stability.

If we write $2B > \sin 2\phi_0(C\rho_0 + A)$ and replace $\sin 2\phi_0$ by its value, we obtain a contradiction, $C\rho_0 > 2C\rho_0$, which shows that the first square bracket in (15.2) is negative and, hence, the first term in (15.2) is positive.

We impose on the second bracket the desired condition $\cos 2\phi_0 < 3E/2C$. Substituting for $\cos 2\phi_0$ its value, one finds $C + A/\rho_0 > C$ which is always fulfilled, so that the second bracket and, hence the second term in (15.2), are negative. The condition of stability is then

$$\sin 2\phi_0(C\rho_0 + A)[\sin 2\phi_0(C\rho_0 + A) - 2B]$$
$$< \tfrac{1}{2}C\rho_0(C\rho_0 + 2A)\cos 2\phi_0(3E/2C - \cos 2\phi_0), \quad (15.3)$$

where all factors are now positive. This inequality is strengthened by writing $(C\rho_0 + A)$ instead of $(C\rho_0 + 2A)$ on the right side. Cancelling $(C\rho_0 + A)$ on both sides and replacing $\sin 2\phi_0$ and $\cos 2\phi_0$ by their values, (15.1), one has

$$(64B^2C^2/(C\rho_0 + 2A)^2) < (C\rho_0 + 2A)^2E^2/(C\rho_0 + A)^2.$$

We strengthen the inequality again by replacing $(C\rho_0 + A)^2$ by $(C\rho_0 + 2A)^2$ so that the inequality yields finally

$$\rho_0 > (2/3E)(4B - 3A). \quad (15.4)$$

If $B < 3A/4$, any positive real root ρ_0 of (4.2) is certainly stable. On the other hand, in section 8 it was shown that if $B < A/2$, the system represented by (1.1) is self-excited. We conclude, therefore, that the condition

$$B < A/2 \tag{15.5}$$

is *sufficient* for the existence of a stable stationary oscillation.

In the experimental arrangement of the pendulum the damping coefficient is generally very small so that (15.5) is always fulfilled and this explains the mechanism of the maintenance of the oscillation by alternating current.

16. ASYMMETRICAL CASE

In the asymmetrical case the function $L(\theta)$ is of the form

$$L(\theta) = L_0 + \alpha_1\theta + \alpha_3\theta^3. \tag{16.1}$$

Rocard uses this form with $\alpha_3 = 0$. In this case L_0 is a value of $L(\theta)$ for $\theta = 0$ of the pendulum.

If one carries out the calculations similar to those outlined in sections 12 and 13, one obtains the following expression for i:

$$i = i_0 \cos\omega t + i_1 \cos(\omega + \Omega)t + i_1' \cos(\omega - \Omega)t + i_3 \cos(\omega + 3\Omega)t + \cdots, \tag{16.2}$$

and, instead of (13.2),

$$i^2 = q_0' + q_1 \cos\Omega t + q_3 \cos 3\Omega t + \cdots. \tag{16.3}$$

The coupling term in this case is

$$d/d\theta(\tfrac{1}{2}Li^2) = -\tfrac{1}{2}(q_0'\alpha_1 + q_1(\alpha_1 + 3\alpha_3\theta^2)\cos\Omega t + q_3(\alpha_1 + 3\alpha_3\theta^2)\cos 3\Omega t) \cdots. \tag{16.4}$$

After re-grouping the terms, (11.2) becomes

$$J\ddot{\theta} + D\dot{\theta} + C\theta + \tfrac{3}{2}\alpha_3\theta^2(q_0' + q_1\cos\Omega t + q_3\cos 3\Omega t) = -q_0' - q_1\alpha_1\cos\Omega t - q_3\alpha_1\cos 3\Omega t. \tag{16.5}$$

The left hand side of this differential equation contains, in addition to the usual pendulum terms, a nonlinear term multiplied by trigonometric terms. These terms have thus the appearance of the parametric terms but in this case, as is easy to show, following the procedure of section 14, that they do not lead to any parametric action. In fact, the calculation shows that the system behaves as an ordinary dissipative system in spite of the presence of terms with periodic coefficients.

It remains to investigate the effect of the right hand terms in (16.5). These terms contain α_1 in their coefficients and are, therefore, inherent in the form $L(\theta)$ considered by Rocard. The first constant term $-q_0'\alpha_1$ merely displaces the zero point of the oscillation as is to be expected in the asymmetrical case. The last term with frequency $3\Omega t$ obviously

does not affect the pendulum so that there remains only the term $-q_1\alpha_1 \cos\Omega t$ capable of influencing the pendulum.

Since q_1 as well as α_1 are small quantities, this effect is of the second order and can be treated by one of the methods of approximation of the nonlinear theory.

17. CONCLUDING REMARKS

It is useful to supplement the preceding study by a few remarks concerning the physical significance of parameters b, a, c, and e (or B, A, C, and E in (3.7) and (3.8)). In section 1, we have introduced somewhat intuitive definitions of "active" and "passive" terms in connection with (1.1), considered as representing an oscillator. The name of the former terms referred to their property of causing an increase in the amplitude and the name of the latter terms referred to their property of limiting the amplitude.

From this point of view, the parameters B and $E > 0$ are passive since they contribute to the limitation of amplitude of the oscillatory process, the first through the dissipation of energy, and the second by bringing into play the "nonlinear elasticity" of the system.

The parameter A, on the contrary, is an active one in this terminology. In fact, we saw that whenever $A = 0$, the system ceases to be self-excited and, hence, no stable stationary oscillation exists. This occurs in the following particular cases studied previously:

Cases (5) and (6):	$\ddot{x} + b\dot{x} + (1 - cx^2 \cos 2t)x \pm ex^3 = 0,$
Case (11):	$\ddot{x} + b\dot{x} + (1 - cx^2 \cos 2t)x = 0,$
Cases (12) and (13):	$\ddot{x} \quad + (1 - cx^2 \cos 2t)x \pm ex^3 = 0,$
Case (14):	$\ddot{x} \quad + (1 - cx^2 \cos 2t)x = 0.$

The reason for this is that the term $(1 - cx^2 \cos 2t)x$ appearing in these differential equations *is not* an active term (that is, is not of the Mathieu, or "parametric" type) so that no energy is introduced into the system through this term. The condition $A < 0$, as was shown, does not rule out the existence of a stable stationary oscillation in general. *It is the condition $A = 0$ which rules out this existence.*

As regards the parameter C, it may be either active or passive according to the circumstances. If C has the same sign as A, it is an active term; if it is of an opposite sign, it is a passive term. If $C = 0$, the process cannot be limited by the dissipative term with B, as is well known from the theory of linear differential equations with periodic coefficients (1, pp. 96 and 97).

The question of the limitation of amplitudes (and hence the possibility of a stationary solution) reduces thus to two typical cases: $E \neq 0$, $C = 0$ (we assume that C is a passive coefficient) and $E = 0$, $C \neq 0$ which characterize the presence of only one nonlinear passive term in

the differential equation. The final result in both cases is the same, the limitation of the amplitude of the oscillatory process, but the mechanism through which this limitation operates is different in both cases.

In the first case (7.1) we have a *linear* Mathieu differential equation supplemented by a nonlinear term ex^3. Obviously, for $E = 0$, this degenerates into the usual Mathieu differential equation in its unstable zone so that no stable stationary oscillation is possible.

In the second case (6.15*a*) the situation is different. Here we have an essentially nonlinear Mathieu differential equation with *a moving parametric point*. In fact, for $x \cong 0$ this is practically a linear Mathieu differential equation with a parametric point $(1, a)$ in the unstable zone, which accounts for the initial increase of the amplitude (that is, the self-excitation). As x increases, the parametric point recedes into the region of the decreasing characteristic exponents (1, Fig. 11, p. 98), and intuitively one may expect that the amplitude becomes stationary when the parametric point reaches the curve of a zero characteristic exponent when the motion becomes periodic (4).

If, however, C is of the same sign as A, this migration of the parametric point occurs in the direction of increasing characteristic exponents, so that no stable stationary oscillation is possible under this condition.

Although this intuitive argument based on the Mathieu function theory leads qualitatively to a correct result, it is impossible to proceed further on this basis inasmuch as during this transition the phenomenon is governed not by one Mathieu equation but by an infinity of such equations as the parametric point moves continuously in the parameter plane parallel to the axis q of the general theory.

The justification of this point, on the contrary, is very simple on the basis of the "stroboscopic" differential equations (3.7) and (3.8) and leads to the limit $\rho_0 = A/C$ at which point the Mathieu oscillator becomes a harmonic oscillator, at least in the first approximation. In the approximations of higher orders small fluctuations, similar to those which have been investigated in an earlier paper (5), appear.

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THE EARTH AND THE FULNESS THEREOF *

BY

FIRMAN E. BEAR[1]

Soil, like faith, is the substance of things hoped for, the evidence of things not seen. Some 2½ billion people depend on it for food and clothing. And, by and large, it never fails. Here and there, because of ignorance and inefficiency, mingled with war and pestilence, a few millions may starve, and other millions may go to bed hungry more or less regularly. But the productive capacity of the earth as a whole is far greater than is required to feed all the people on the earth.

Nevertheless, the Malthusian principle that population is kept under control by famine, pestilence, and war still applies over a large part of the earth. It remains to be determined whether this principle can be permanently set aside by man through the application of the sciences at his command.

In a recent book on "Human Fertility, the Modern Dilemma," Robert C. Cook estimates that world population is increasing at the rate of 68,000 every 24 hours. This is equivalent to 25 million, or 1 per cent, a year. The rate of population growth in the United States is higher than the average for the rest of the earth.

When Captain John Smith first set foot on the shore of Virginia in 1607, the 1903 million acres of land that now constitute the United States of America were inhabited by some 800,000 Indians. Each of these Indians had 2400 acres of land at his disposal. Over this the braves hunted, fished, and fought, while the squaws did the work. This was an Indian Paradise. Today, only 345 years later, our population is 156 million, and the amount of land for each person has been reduced to 12 acres. Over half of this is desert, mountain, lake, or swamp.

How long is 345 years? As a small boy, I remember talking with my great grandfather, who was then 94 years old. Through his eyes I can now look back nearly half that period of time. This is a very young country. We have come a long way in a very short time. But we have had time enough to lay waste a lot of forest and to ruin a lot of land that we inherited from the Indians. This has been a land of plenty, but we are catching up with the food supply. And 2 million more people join us at the table every year.

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Previous to their coming to America our European ancestors had never seen corn, potatoes, tobacco, peanuts, nor tomatoes. They tasted these Indian crops, liked them, and soon set about growing them. Cotton, another crop with which they had had no experience, grew well in the South. In a surprisingly short time the farmers of this country were growing 150 million acres of these crops every year. They plowed the land, planted the crops in long, straight rows, cultivated between the rows, and developed the most destructive type of agriculture the world has ever known.

Our people are mechanical minded. The 2000 tractors of 1910 grew to 3½ million faster-moving types by 1950. The horse and mule population fell from 25 million to 7½ million. Fences, the Maginot line against erosion, were torn out to make more room for the tractors. This gave the wind and water a longer sweep. As a result millions of tons of good topsoil were carried down the slopes and dumped into the rivers. Mud clogged the harbors. Silt poured into the reservoirs, reducing their storage capacity from 1 to 3 per cent a year. And, beginning in early 1934, dust storms tore the soil loose from its moorings in Kansas wheat fields, spread it over the country to the east, and carried some of it on out to sea.

The handwriting of erosion was plain for everyone to see. People in high places began to be cognizant of the damage that was being done. Public sentiment was aroused. The Soil Conservation Act was passed in 1935, and the Soil Conservation Service came into existence. Strip-cropping, contour-farming, the growing of more sod-crops, the seeding of more cover crops, and the planting of forests got under way. Farmers began to realize that it was necessary to take greater advantage of roots as a means of sewing the soil to the earth. But millions of acres of good land had been ruined before this came to pass.

In 1607 virtually all the land east of the Mississippi River, except for the prairies of northern Indiana and Illinois, was covered with forest. A squirrel could have gone from Maine to Louisiana without touching the ground, merely by jumping from one giant tree to the next. These trees were in our way. So we cut them down, burned them up, laid bare the land, and put it to the plow.

Judging by the luxuriance of these forests, one would have thought that the soil beneath them was highly productive. For the moment, it was. But once the trees had been cleared off and the soil had been brought under cultivation a large part of the accumulated organic matter was soon used up. Then we discovered that the underlying soil had lost virtually all its soluble substance to the sea. Limestone, phosphate, and potash salts had to be supplied. Last year American farmers applied nearly 50 million tons of these soil amendments. This is enough to fill a line of 40-ton freight cars extending nearly 4 times the distance from New York to San Francisco.

The farther west one goes in the United States the less the rainfall and the smaller the amount of soluble material that has been washed out of the soil. In the desert areas soils are highly productive, once irrigation has been applied. Some 25 million acres of desert land are being irrigated, and there is water for some 20 million more. East of the 150 million acres that constitute the Great American Desert is a 600 million-acre area of semi-desert with a rainfall of 10 to 20 inches. Some 300 million additional acres are contained in the sub-humid portion of the Great Plains. All these regions need water rather than lime or fertilizer.

Our crop land for 1950 totaled 358 million acres. Included in this is a vast area of rich black soil that extends from the Dakotas south to Texas. This land needs phosphate and potash, but not lime. East of this black-land area, where the rainfall is heavier, both lime and fertilizer have to be used in large amounts. By and large, it is easier to haul the necessary salts back from ancient seas and put them on the land from which they came than it is to provide the much larger tonnages of water that are required to bring more of the desert under production.

But the problem of maintaining adequate supplies of nutrient elements in the soils of humid regions becomes more and more troublesome as the years go by. Virgin fertility has largely been exhausted. We are no longer satisfied with the yields that unaided Nature can provide. The modern goal is 100 bushels of corn, 50 bushels of wheat, 500 bushels of potatoes, 20 tons of tomatoes, and 5 tons of alfalfa hay an acre.

As the soil becomes older agriculturally and the yields are stepped up by growing improved hybrids and disease-resistant varieties, by using improved tillage implements, and by applying more lime and NPK fertilizer, other deficiencies develop. In certain areas magnesium must be supplied. In other areas sulfur is needed. Under some circumstances iron deficiencies develop. In still others the limiting factor in plant and animal production is a lack of boron, molybdenum, copper, manganese, zinc, cobalt, or iodine. These last seven trace elements present some of our most troublesome problems.

Take boron, for example. Over 12 per cent of the soil of New Jersey is known to be in need of this element for maximum productiveness. Field tests with alfalfa have shown that a 25-pound application of borax an acre may make all the difference between success and failure. It is not uncommon to be able to increase yields of hay as much as 1 ton an acre by applying 75 cents' worth of borax.

More recently we have been intrigued by the possibility of widespread need for molybdenum. In the beginning phases of our work on this element on alfalfa we had great difficulty developing a deficiency in plants grown in sand culture. Finally, by redistilling the water and

recrystallizing all the nutrient salts, deficiency symptoms were developed in sand cultures. The plants stopped growing and turned yellow. But all that was required to start them on again was 1 part molybdenum in 1 billion parts of the nutrient solution. Yet year before last a New Jersey vegetable grower plowed under 45 acres of cauliflower that couldn't produce heads because the soil lacked available molybdenum.

The soil of this great land of ours is capable of producing food in abundance for possibly 1 billion people. Hopefully, there will never be that many people to feed. But if the time comes that there are, much more of the total expendable budget will have to be invested in food. Some outlying land will have to be reclaimed from the sea, as the Dutch have done it. Additional water for irrigation will have to be distilled from the ocean. Tremendous tonnages of lime and fertilizer will have to be used. Floods will have to be brought under complete control. The soil on every acre of land will have to be conserved. The problems of growing enough food for all these people will be very serious ones. We will have to be satisfied with less meat and more bread, potatoes, and vegetables. Some of our less fortunate folks may go to bed hungry, as do millions of people every night in the more densely populated parts of the earth.

The soil scientists of America, of which I am proud to be one, are dedicated to the task of conserving the Nation's soil, water, and life resources to the end that, whatever the final population may be, the issue can be met. We feel under obligation to leave no stone unturned to the end that our land is conserved and its productivity is not only maintained but raised to ever higher levels. Our aim is to see to it that our increasing numbers of fine young folks continue to enjoy the fruits of the earth in abundance.

NETWORK ANALYSIS BY LEAST POWER THEOREMS *

BY

FREDERICK L. RYDER [1]

SUMMARY

There are discussed electrical power theorems, analogous to certain structural theorems based on work of deformation, which permit the solution of the general linear network without the use of Kirchhoff's second law, in the mesh-analysis case, and of Kirchhoff's first law, in the nodal-analysis case. While well-known for the d-c. resistive case, these theorems are here placed in a form considered more suitable for practical network analysis, and are extended to a-c. networks containing transformers and complex impedances. The resulting procedures for network analysis are more direct than conventional methods in networks containing transformers.

INTRODUCTION

The purpose of this paper is to develop methods for solving linear electrical networks by making use of certain relationships involving the electric power. Analogous methods based on work of deformation have long been a conventional and powerful tool in the analysis of statically loaded structures, and are embodied in the following two theorems (1):[2]

1. The *theorem of least work*, which states that the redundant forces and moments (those which cannot be evaluated by considerations of equilibrium) assume such values as to minimize the total work of deformation of the structure. This enables one to solve for the redundants without regard to deflections or slopes, in the following manner: the work W is written as a function of the applied forces and moments, the redundants, and the elastic constants, its partial derivative with respect to each redundant then being set to zero. In physically realizable structures the resulting equations always suffice to yield the solution.

2. *Castigliano's theorem*, which states that

$$D' = \partial W / \partial F, \tag{1}$$

where F is any applied force, and D' is the component of deflection, measured parallel to the direction of F, of the point at which F is applied to the structure. If all internal forces are known in terms of applied forces, then W is known in terms of the latter, and Eq. 1 will yield the deflection component associated with each applied force. A similar relationship can be written to relate slope components to applied moments.

* This paper is based on a thesis (see reference 5) which has been accepted by the faculty of the Graduate Division of the College of Engineering of New York University in partial fulfillment of the requirements for the degree of Doctor of Engineering Science.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Castigliano's theorem and the theorem of least work are sometimes called Castigliano's first and second theorems, respectively.

Electrical theorems analogous to the theorem of least work have been demonstrated for the d-c. resistive case by James Jeans (2) and by Black and Southwell (3, 4). The latter authors discuss the structural-electrical analogy in some detail, with particular regard to relaxation methods of numerical approximation. The present paper will extend these theorems to a-c. networks containing transformers and complex impedances, will develop electrical equivalents of Castigliano's theorem, and will discuss applications to practical network analysis, with the aid of several examples.

THEOREMS OF JEANS AND OF BLACK AND SOUTHWELL

The theorems of Jeans and of Black and Southwell will first be presented in a form rather different from that used by these authors, but which is thought to lead more directly to practical applications in network analysis.

Jeans' Theorem

In any portion of a linear network of resistors and d-c. voltage sources, such as is shown in Fig. 1, let the mesh currents be $I_1, I_2, \ldots$.

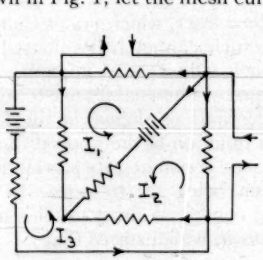

FIG. 1.

The current, associated with each mesh current, flowing in a specified reference direction (indicated by an arrow) in any branch j is $b_{j1}I_1$, $b_{j2}I_2$, . . . , where b is either $+1$ or -1. Write the following function of the electrical power:

$$P = \sum_j [b_{j1}I_1 + b_{j2}I_2 + \cdots]^2 R_j - 2\sum_j V_j[b_{j1}I_1 + b_{j2}I_2 + \cdots], \quad (2)$$

where R_j is branch resistance and V_j is the rise of the branch voltage source, if any, measured positive in the reference direction. The expression within the brackets is the branch current flowing in the reference direction. The first term is the power dissipated in the resistors,

and the second term, including the minus sign, is twice the power absorbed by the voltage sources.

Let one of the mesh currents, say I_1, be increased by the quantity ΔI_1, not necessarily small. This does not violate Kirchhoff's first law, that of current-continuity. For a single branch j the power function becomes

$$p_j + \Delta p_j = [b_{j1}I_1 + b_{j2}I_2 + \cdots + b_{j1}\Delta I_1]^2 R_j - 2V_j[b_{j1}I_1 + b_{j2}I_2 + \cdots + b_{j1}\Delta I_1].$$

Comparing this equation with the previous one, we find that

$$\Delta p_j = \overline{b_{j1}\Delta I_1}^2 R_j + 2b_{j1}\Delta I_1[(b_{j1}I_1 + b_{j2}I_2 + \cdots)R_j - V_j].$$

The quantity within the brackets is total voltage drop, measured in the reference direction. The summation of this expression over the network is taken around the mesh associated with I_1, the branch coefficient b_{j1} being equal to -1 when the reference branch direction is opposed to the direction of traverse of the mesh, as is apparent in Fig. 1. Hence the summation of the second term is zero, and, since Δp is positive, P must be a minimum subject to Kirchhoff's first law.

This result may be called the *theorem of least power*, and can be used instead of Kirchhoff's second law (that of mesh summation of voltage) to effect a solution. This is done by successively setting to zero the partial derivative of P with respect to each of the unknown mesh currents, these being considered as *redundant*.

The method can be compared to that based on the structural theorem of least work, whence it is seen that the requirement of current-continuity, implied in drawing the mesh currents, is analogous to the requirement of equilibrium of forces and moments.

Certain obvious extensions of Jeans' theorem can be made at once. For example, to derive a theorem analogous to that of Castigliano, extend the network being considered in Fig. 1 to include a resistanceless branch, carrying an independently supplied current I_x, which has for its end-points two nodes whose voltage difference is sought. The voltage rise of this branch is V_x, both current and voltage rise being measured in a reference direction. Considering I_x as an additional mesh current, we find the power function to be

$$P_x - 2V_xI_x, \tag{3}$$

where

$$P_x = \sum_j [b_{j1}I_1 + b_{j2}I_2 + \cdots + b_{jx}I_x]^2 R_j - 2\sum_j V_j[b_{j1}I_1 + b_{j2}I_2 + \cdots + b_{jx}I_x], \tag{4}$$

the summation over j not including x. On setting to zero the partial

derivative of the power function with respect to I_x, we find

$$V_x = \tfrac{1}{2}\partial P_x/\partial I_x, \tag{5}$$

so that V_x can be determined without the aid of Kirchhoff's second law. The numerical value of I_x must not be inserted until after the differentiation is performed. This value is frequently zero, since the branch associated with I_x is often created just for the purpose of evaluating V_x.

The factor $\frac{1}{2}$ in Eq. 5 is seen to be lacking in Eq. 1. This can be shown to arise from the fact that the work associated with a force F and its deflection component D' is $FD'/2$, while the corresponding electric power is simply IV.

The extension of the above theorems to the constant-phase a-c. case (180° phase shift being replaced by a minus sign) is immediate, providing rms values are used for current and voltage, and average rather than instantaneous power is considered.

Finally, the theorems can be extended to include networks containing ideal transformers, such networks being called "reactanceless" to distinguish them from the purely resistive type. An ideal transformer may be defined as one which satisfies the following requirements:

$$\left.\begin{aligned} V_s &= NV_p, \\ I_s &= -I_p/N, \end{aligned}\right\} \tag{6}$$

where p and s stand for primary and secondary, respectively, N is the nominal turns ratio and may be negative, and V is transformer voltage rise, measured in the direction of current. Practical transformers can be closely simulated by well-known conventional networks which include an ideal transformer. The corresponding terms in Eq. 2 are

$$-2(V_pI_p + V_sI_s),$$

which is zero, by Eq. 6. Hence these terms can be ignored in writing the expression for the power function.

In networks containing ideal transformers Kirchhoff's first law must here be extended to fix the relationship between primary and secondary currents, according to the turns ratios. Hence, a particular mesh current may be considered to traverse more than one mesh, and the associated b's in Eq. 2 may have any values, depending on the turns ratios.

Theorem of Black and Southwell

For the constant-phase reactanceless network, let each end of each resistor, transformer winding, and voltage source be considered a node, numbered 1, 2, . . . n, ... The node voltages are V_n, measured above some convenient datum, and are considered to be linearly related according to Kirchhoff's second law, that of voltage summation, only

insofar as these relationships are fixed by voltage sources and transformer turns ratios. Thus, in general,

$$V_n = c_{n1}V_1 + c_{n2}V_2 + \cdots, \tag{7}$$

where the voltages on the right, which may include known voltage sources, are called the *independent* voltages, and are so chosen that all node voltages can be expressed in terms of them by Kirchhoff's second law. The unknown independent voltages are considered to be *redundant*. Write the following function of electrical power:

$$Q = \sum_n \{[(V_n - V_1)^2 G_{1n} - 2(V_n - V_1)I_{1n}] + [(V_n - V_2)^2 G_{2n} - 2(V_n - V_2)I_{2n}] + \cdots\},$$

where G is conductance ($= 1/R$), and I_{1n}, for example, is current flowing from 1 to n other than that flowing through G_{1n}. The first term in each bracket is power loss in the conductance, and the second term, including the minus sign, is twice the power absorbed due to the flow of I between the indicated nodes. I may be a transformer winding current, or a current flowing through a voltage or current source.

Let any one of the redundants, say V_1, be increased by the quantity ΔV_1, not necessarily small. Using Eq. 7, we find the power function for a single node n to be

$$\begin{aligned} q_n + \Delta q_n = {} & [V_n - V_1 + \Delta V_1(c_{n1} - c_{11})]^2 G_{1n} \\ & - 2[V_n - V_1 + \Delta V_1(c_{n1} - c_{11})]I_{1n} \\ & + [V_n - V_2 + \Delta V_1(c_{n1} - c_{21})]^2 G_{2n} \\ & - 2[V_n - V_2 + \Delta V_1(c_{n1} - c_{21})]I_{2n} + \cdots. \end{aligned}$$

Expanding this, and subtracting the value of q_n, we get

$$\begin{aligned} \Delta q_n = {} & \overline{\Delta V_1}^2[(c_{n1} - c_{11})^2 G_{1n} + (c_{n1} - c_{21})^2 G_{2n} + \cdots] \\ & + 2\Delta V_1\{(c_{n1} - c_{11})[(V_n - V_1)G_{1n} - I_{1n}] \\ & + (c_{n1} - c_{21})[(V_n - V_2)G_{2n} - I_{2n}] + \cdots\}. \end{aligned}$$

The first term is positive. In the second term, the first bracket, for example, represents the total current flowing from n to 1. The total coefficient of c_{n1} within the braces is seen to be the total current leaving n for other nodes, which is zero, by Kirchhoff's first law. Similarly, the coefficient of c_{21}, for example, when summed over n, represents the total current flowing from 2 to all other nodes, which is also zero. Therefore, the second term is zero, Δq is positive, and Q must be a minimum, subject to Kirchhoff's second law as affected by transformer turns ratios and voltage sources.

This is the *dual* of Jeans' theorem, current and voltage being interchanged. It can be used to solve for the redundants in Eq. 7 without

regard to Kirchhoff's first law, by setting to zero the partial derivative of Q with respect to each redundant in turn.

To develop the corresponding electrical equivalent of Castigliano's theorem, add to the network a fixed voltage source V_x, and let it be required to find I_x, the current through V_x in the direction of voltage rise. With V_x considered as one of the independent voltages in Eq. 7, the power function becomes

$$Q_x - 4V_xI_x, \tag{8}$$

where Q_x is the same as Q except that V_n is modified because of V_x, and the summation over n does not include the x-branch. On setting to zero the partial derivative of the power function with respect to V_x, we find

$$I_x = \tfrac{1}{4}\partial Q_x/\partial V_x.$$

The factor $\frac{1}{4}$ replaces the factor $\frac{1}{2}$ in Eq. 5 because each branch is counted twice in the present summation. Thus I_x can be determined without the aid of Kirchhoff's first law.

If ideal transformers exist, the sum of the power function terms for each transformer is twice that for the corresponding terms in the power function P, which was shown to be zero. Hence these terms can be ignored in writing Q.

EXTENSION TO COMPLEX A-C. NETWORKS

Jeans' Theorem

For any branch j, the sum of resistive power dissipation and twice the power absorbed by the voltage sources is

$$p_j = [b_{j1}\mathbf{I}_1 + b_{j2}\mathbf{I}_2 + \cdots] \cdot [(b_{j1}\mathbf{I}_1 + b_{j2}\mathbf{I}_2 + \cdots)R_j - 2\mathbf{V}_j], \tag{9}$$

where the b's are scalar, $\mathbf{I} = I\underline{|\theta}$, $\mathbf{V} = V\underline{|\varphi}$, and the dot denotes scalar multiplication. The magnitudes I and V may be negative, a minus sign denoting a phase shift of 180°. This expression is independent of any reactances which may be present.

Let one redundant mesh current, say $\mathbf{I}_1$, be increased by the vector increment $\Delta\mathbf{I}_1$, here considered very small in order to simplify the analysis. This yields

$$p_j + \Delta p_j = [b_{j1}(\mathbf{I}_1 + \Delta\mathbf{I}_1) + b_{j2}\mathbf{I}_2 + \cdots] \cdot \{[b_{j1}(\mathbf{I}_1 + \Delta\mathbf{I}_1) + b_{j2}\mathbf{I}_2 + \cdots]R_j - 2\mathbf{V}_j\}.$$

Expanding, subtracting p_j, and ignoring infinitesimals of the second order and higher, we find

$$\Delta p_j = 2b_{j1}\Delta\mathbf{I}_1 \cdot [(b_{j1}\mathbf{I}_1 + b_{j2}\mathbf{I}_2 + \cdots)R_j - \mathbf{V}_j].$$

To evaluate this, we replace $\Delta\mathbf{I}_1$ by its components $\Delta I_1\underline{|\theta_1}$ and $I_1\Delta\theta_1\underline{|\theta_1 + 90°}$. On performing the scalar multiplication, there is

obtained

$$\Delta p_j = 2b_{j1}\Delta I_1\{[b_{j1}I_1 + b_{j2}I_2\cos(\theta_2 - \theta_1) + \cdots]R_j - V_j\cos(\varphi_j - \theta_1)\} + 2b_{j1}I_1\Delta\theta_1\{[b_{j2}I_2\sin(\theta_2 - \theta_1) + b_{j3}I_3\sin(\theta_3 - \theta_1) + \cdots]R_j - V_j\sin(\varphi_j - \theta_1)\}. \quad (10)$$

The term in the first braces is the branch voltage drop, in the reference direction, measured in phase with $\mathbf{I}_1$, less the voltage drop at the same phase across branch reactance X. The term in the second braces is similar, except that it is referred to the phase leading that of $\mathbf{I}_1$ by 90°. The reactive drops are, respectively,

$$-[b_{j2}I_2\sin(\theta_2 - \theta_1) + b_{j3}I_3\sin(\theta_3 - \theta_1) + \cdots]X_j,$$
$$[b_{j1}I_1 + b_{j2}I_2\cos(\theta_2 - \theta_1) + \cdots]X_j.$$

On making use of the fact that the total voltage drop around each mesh is zero, and on letting the increments ΔI_1 and $\Delta\theta_1$ approach zero, there is finally obtained the following:

$$\left.\begin{aligned}\frac{\partial P}{\partial I_1} &= 2\sum_j\sum_k b_{j1}b_{jk}I_k\sin(\theta_k - \theta_1)X_j\\ \frac{\partial P}{I_1\partial\theta_1} &= -2\sum_j\sum_k b_{j1}b_{jk}I_k\cos(\theta_k - \theta_1)X_j,\end{aligned}\right\} \quad (11)$$

where j denotes each branch, k denotes each mesh current, and P is the power function for the entire network under consideration. If the network is reactanceless, the power function is a minimum (or at least its partial derivative is zero) with respect to each redundant magnitude and phase angle. In any case, the redundants can be evaluated by Eq. 11 without the use of Kirchhoff's second law.

The equivalent of Castigliano's theorem for this case can be obtained by writing the power function as

$$P_x - 2\mathbf{V}_x\cdot\mathbf{I}_x = P_x - 2V_xI_x\cos(\varphi_x - \theta_x),$$

which takes the place of expression (3). On differentiating this with respect to I_x and θ_x in turn and making use of Eq. 11, it is found that

$$\left.\begin{aligned}V_x\cos(\varphi_x - \theta_x) &= \frac{1}{2}\frac{\partial P_x}{\partial I_x} - \sum_j\sum_k b_{jx}b_{jk}I_k\sin(\theta_k - \theta_x)X_j\\ V_x\sin(\varphi_x - \theta_x) &= \frac{1}{2}\frac{\partial P_x}{I_x\partial\theta_x} + \sum_j\sum_k b_{jx}b_{jk}I_k\cos(\theta_k - \theta_x)X_j.\end{aligned}\right\} \quad (12)$$

Thus the components of $\mathbf{V}_x$ in phase with and 90° ahead of $\mathbf{I}_x$ may be determined without the aid of Kirchhoff's second law.

By Eq. 9, it is seen that the terms associated with each ideal transformer take the form

$$-2(\mathbf{V}_p\cdot\mathbf{I}_p + \mathbf{V}_s\cdot\mathbf{I}_s).$$

By writing Eq. 6 in suitable vector form (N remaining scalar), these terms are seen to be zero, and so may be omitted from the power function.

Theorem of Black and Southwell

For each node the expression for the power function takes the form

$$q_n = (\mathbf{V}_n - \mathbf{V}_1)\cdot[(\mathbf{V}_n - \mathbf{V}_1)G_{1n} - 2\mathbf{I}_{1n}] + (\mathbf{V}_n - \mathbf{V}_2)\cdot[(\mathbf{V}_n - \mathbf{V}_2)G_{2n} - 2\mathbf{I}_{2n}] + \cdots.$$

This expression is independent of any susceptances which may be present. Let one redundant, say $\mathbf{V}_1$, be increased by the vector increment $\Delta\mathbf{V}_1$, here considered very small. The above expression is increased incrementally according to the vector form of Eq. 7, where the c's remain scalar. Expanding the resulting expression, neglecting infinitesimals of second order and higher, and substracting q_n, we find

$$\Delta q_n = 2(c_{n1} - c_{11})\Delta\mathbf{V}_1\cdot[(\mathbf{V}_n - \mathbf{V}_1)G_{1n} - \mathbf{I}_{1n}] + 2(c_{n1} - c_{21})\Delta\mathbf{V}_1\cdot[(\mathbf{V}_n - \mathbf{V}_2)G_{2n} - \mathbf{I}_{2n}] + \cdots.$$

Replacing $\Delta\mathbf{V}_1$ by its components $\Delta V_1|\underline{\varphi_1}$ and $V_1\Delta\varphi_1|\underline{\varphi_1 + 90°}$, and performing the scalar multiplications, it is found that

$$\Delta q_n = 2\Delta V_1\sum_m (c_{n1} - c_{m1})\{[V_n\cos(\varphi_n - \varphi_1) - V_m\cos(\varphi_m - \varphi_1)]G_{mn} - I_{mn}\cos(\theta_{mn} - \varphi_1)\} + 2V_1\Delta\varphi_1\sum_m (c_{n1} - c_{m1})\{[V_n\sin(\varphi_n - \varphi_1) - V_m\sin(\varphi_m - \varphi_1)]G_{mn} - I_{mn}\sin(\theta_{mn} - \varphi_1)\}, \quad (13)$$

where m is summed over all nodes. The first braces contain the total current, measured in phase with $\mathbf{V}_1$, flowing from node n to node m, except for the corresponding current flowing through the branch susceptance B_{mn}, if any. The second braces contain a similar quantity, referred to a phase leading that of $\mathbf{V}_1$ by 90°. Let us temporarily denote the quantity in either braces by

$$T_{mn} - J_{mn},$$

where T is total current and J is susceptive current. The summation $c_{n1}\sum_m T_{mn}$ is zero, since it is a constant times the total current leaving n for all other nodes. The summation $\sum_m c_{m1}T_{mn}$, when summed over the entire network for a particular value of m, is a constant times the total current from m to all other nodes, and so is also zero. Hence T may be ignored in the summation for the power function Q over the entire network under consideration. Expressing J suitably, and letting the in-

crements approach zero, one finally obtains the following:

$$\left.\begin{aligned}\partial Q/\partial V_1 &= 2\sum_n\sum_m (c_{n1} - c_{m1})[V_n \sin(\varphi_n - \varphi_1) \\ &\qquad - V_m \sin(\varphi_m - \varphi_1)]B_{mn}, \\ \partial Q/V_1\partial\varphi_1 &= -2\sum_n\sum_m (c_{n1} - c_{m1})[V_n \cos(\varphi_n - \varphi_1) \\ &\qquad - V_m \cos(\varphi_m - \varphi_1)]B_{mn}.\end{aligned}\right\} \tag{14}$$

If the network is reactanceless, Q is a minimum (or at least its partial derivative is zero) with respect to each redundant magnitude and phase angle.

To derive the equivalent of Castigliano's theorem, write the power function as

$$Q_x - 4\mathbf{V}_x\cdot\mathbf{I}_x = Q_x - 4V_xI_x\cos(\theta_x - \varphi_x),$$

which takes the place of expression (8). On differentiating this with respect to V_x and φ_x in turn, and making use of Eq. 14, it is found that

$$\left.\begin{aligned}I_x\cos(\theta_x - \phi_x) &= \frac{1}{4}\frac{\partial Q_x}{\partial V_x} - \frac{1}{2}\sum_n\sum_m (c_{nx} - c_{mx}) \\ &\qquad \times[V_n \sin(\varphi_n - \varphi_x) - V_m \sin(\varphi_m - \varphi_x)]B_{mn} \\ I_x\sin(\theta_x - \phi_x) &= \frac{1}{4}\frac{\partial Q_x}{V_x\partial\varphi_x} + \frac{1}{2}\sum_n\sum_m (c_{nx} - c_{mx}) \\ &\qquad \times[V_n \cos(\varphi_n - \varphi_x) - V_m \cos(\varphi_m - \varphi_x)]B_{mn}\end{aligned}\right\}. \tag{15}$$

Lastly, power function terms associated with ideal transformers may be ignored, as the reader can easily show by the methods used in the previous cases.

APPLICATIONS

While the expressions encountered in the power theorems appear formidable, it will be seen that they can be formulated in such a way as to make their application simple.

Modified Mesh Analysis

Consider first the partial derivatives of P as readily constructed from Eq. 10. These contain groups of components both in phase with $\mathbf{I}_1$ and leading $\mathbf{I}_1$ by 90°, as is also true of Eq. 11. By multiplying the quadrature-leading components by the a-c. operator $\sqrt{-1}$ and adding them to the in-phase components, there is obtained, by suitable combination of Eqs. 10 and 11,

$$\sum_j\sum_k b_{j1}(b_{jk}\mathbf{I}_k\mathbf{Z}_j - \mathbf{V}_j) = 0 = \sum_j b_{j1}(\mathbf{I}_j\mathbf{Z}_j - \mathbf{V}_j), \tag{16}$$

where $\mathbf{Z}$ is complex impedance. If no transformers are present, this is

nothing more or less than Kirchhoff's second law as applied in the conventional mesh analysis, the b's merely adjusting for the direction of traverse of the mesh with respect to the branch reference directions. Hence in this case the power method offers no advantage over the usual mesh analysis.

On the other hand, if transformers are present, Eq. 16 offers a *modified mesh voltage summation* which automatically and implicitly takes the unknown transformer voltage rises into account, so that it is unnecessary to consider them specifically. Hence the unknowns are reduced by one voltage per transformer (the two voltages being related by the turns ratio), as compared with the conventional mesh analysis. In complicated networks involving transformers, such as are involved in the electrical analogues of structures in reference 5, this advantage may be considerable.

With currents evaluated, the node voltages can be evaluated by summing voltage along desired paths, if no transformers are present. However, if transformer windings lie in the selected paths, then the corresponding transformer rises must be evaluated, in the ordinary mesh analysis. The necessity for this may be avoided by use of Eq. 12, which can be combined into the following form, by the same method as before:

$$\mathbf{V}_x = \sum_j \sum_k b_{jx}(b_{jk}\mathbf{I}_k\mathbf{Z}_j - \mathbf{V}_j) = \sum_j b_{jx}(\mathbf{I}_j\mathbf{Z}_j - \mathbf{V}_j), \tag{17}$$

where the summation over j does not include x.

The method may be summed up as follows:

1. Select the mesh currents $\mathbf{I}_k$, and evaluate the b's according to the equation

$$\mathbf{I}_j = \sum_k b_{jk}\mathbf{I}_k, \tag{18}$$

where $\mathbf{I}_j$ is branch current in a selected reference direction. This is often convenient to do analytically, using Kirchhoff's first law, and taking account of the turns ratios of ideal transformers.

2. Determine the redundants, or unknown mesh currents, by summations (16), which reduce to the conventional mesh summations if no transformers are present.

3. Determine unknown node voltage differences by summation (17), which reduces to a conventional summation if no transformers are present.

Values of $\mathbf{V}_j$ in Eqs. 16 and 17 associated with ideal transformers may be ignored.

Modified Nodal Analysis

Using the technique of the previous case, we combine Eqs. 13, 14,

and 15 to form

$$\sum_n \sum_m (c_{n1} - c_{m1})[(\mathbf{V}_n - \mathbf{V}_m)\mathbf{Y}_{mn} - \mathbf{I}_{mn}] = 0 \tag{19}$$

$$\mathbf{I}_x = \tfrac{1}{2} \sum_n \sum_m (c_{nx} - c_{mx})[(\mathbf{V}_n - \mathbf{V}_m)\mathbf{Y}_{mn} - \mathbf{I}_{mn}], \tag{20}$$

where $\mathbf{Y}$ is complex admittance, and the summations over m and n do not include the branch x. If no transformers or fixed voltage sources are present, then c equals unity when both subscripts are similar, and otherwise equals zero. Equations 19 and 20 can be shown to lead to conventional nodal summations of current. In the general case, however, Eq. 19 provides a *modified nodal current summation* which takes transformer currents into account implicitly, so that it is unnecessary to consider them. Thus the unknowns are reduced by one current per transformer, the two transformer currents being related by the turns ratio.

With node voltages evaluated by Eq. 19, the current supplied to a particular node can readily be evaluated as the sum of currents leaving that node, if no transformers or fixed voltage sources are present. In the general case, the necessity for separate evaluation of the transformer currents can be avoided by use of Eq. 20.

The method may be summed up as follows:

1. Write the equations

$$\mathbf{V}_n = \sum_d c_{nd}\mathbf{V}_d, \tag{21}$$

where the voltages on the right are considered *independent*, and consist of node voltages and fixed voltage sources in terms of which all node voltages can be evaluated by Kirchhoff's second law, insofar as it is affected by transformer turns ratios and voltage sources.

2. Determine the redundants, or unknown independent voltages, by summations (19).

3. Determine unknown supplied currents by summations (20).

Transformer currents I in Eqs. 19 and 20 may be ignored.

Following the usage in structural analysis, the network may be called *determinate* if no redundants exist, in which case it is solved by use of only one of Kirchhoff's laws. The number of redundants indicate the *degree of indeterminacy* of the network, and is a rough indication of the difficulty of solution. For a given problem the degree of indeterminacy may be quite different for the modified mesh and nodal analyses, the choice between these two methods being made accordingly.

EXAMPLES

In the network of Fig. 2 all branch impedances are $R(1 + j)$ (where $j = \sqrt{-1}$), and the transformer turns ratios are such that $\mathbf{V}_{12} = 2\mathbf{V}_{15}$

and $\mathbf{V}_{32} = 2\mathbf{V}_{36}$. A current $\mathbf{I}_{74} = 2jI$ is generated in branch 74, and a voltage $\mathbf{V}_2 = IR = V$ is applied at node 2, the datum for voltage being taken at node 4. Find the unknown currents and node voltages.

Modified Mesh Analysis

Using Kirchhoff's first law, we can express the branch currents in terms of a single redundant, $\mathbf{I}_a$. For example, to find $\mathbf{I}_{15}$ and $\mathbf{I}_{41}$ in terms of $\mathbf{I}_{21}$, which is assumed equal to $\mathbf{I}_a$, we write

$$\mathbf{I}_{15} = 2\mathbf{I}_a,$$

which is required by the turns ratio, and

$$\mathbf{I}_a + \mathbf{I}_{41} - \mathbf{I}_{15} = 0,$$

as required by current-continuity. These yield the branch currents indicated.

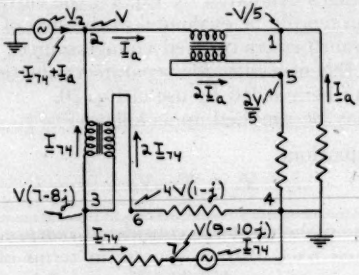

FIG. 2.

The coefficients b_{j1} in Eq. 16 are simply the multiplier of $\mathbf{I}_a$ in each branch, so that this equation yields

$$1(\mathbf{I}_a\mathbf{Z}_{41}) + 2(2\mathbf{I}_a\mathbf{Z}_{54}) + 1(-\mathbf{V}_2) = 0.$$

Substituting the values of $\mathbf{Z}$ and $\mathbf{V}_2$ and solving for $\mathbf{I}_a$, we find

$$\mathbf{I}_a = \frac{I}{10}(1 - j),$$

in terms of which all currents are readily evaluated.

The voltage at node 7 cannot be evaluated by a conventional voltage summation without first evaluating the voltage rises associated with the transformer at node 3. This can be avoided by use of Eq. 17, where b_{j2} is simply the multiplier of $\mathbf{I}_{74}$ in each branch. (For this purpose $\mathbf{I}_{74}$ must be kept in literal, rather than numerical, form.) Taking account

of the reference direction of $\mathbf{I}_{74}$, we have

$$-\mathbf{V}_7 = 1(\mathbf{I}_{74}\mathbf{Z}_{37}) + 2(2\mathbf{I}_{74}\mathbf{Z}_{46}) - 1(-\mathbf{V}_2),$$

which, when evaluated, yields

$$\mathbf{V}_7 = V(9 - 10j).$$

The voltage $\mathbf{V}_{32}$ could be calculated by similar means, by introducing a fictitious current generator between nodes 2 and 3, rewriting the branch currents to include the new current, and using Eq. 17, after which the new current is set to zero.

The transformer voltage rises may be calculated by conventional voltage summations, if desired. These lead to the node voltage values indicated in Fig. 2. With these values the entire solution can be rapidly checked for conformity with Kirchhoff's second law.

Modified Nodal Analysis

Figure 3 represents the same network as Fig. 2, but with the node voltages expressed in terms of the four independent voltages $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_3$,

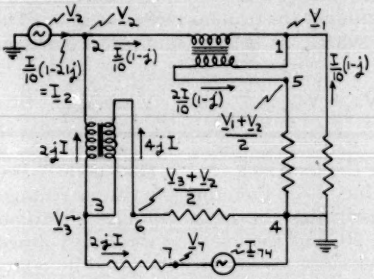

FIG. 3.

and $\mathbf{V}_7$ according to Kirchhoff's second law, insofar as it is affected by transformer voltage rises and fixed voltage sources. For example,

$$\mathbf{V}_6 = \mathbf{V}_1 + \mathbf{V}_{16} = \mathbf{V}_1 + \frac{\mathbf{V}_2 - \mathbf{V}_1}{2} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{2}.$$

Three of the independent voltages are redundant, so that the network is indeterminate to the third degree, as compared with the first degree in the previous analysis. The impedances are now expressed as admittances, and therefore take the form

$$\frac{1}{2R}(1 - j).$$

The coefficients c in Eq. 19 are simply the multiplier of the appropriate redundant at each node. For $\mathbf{V}_7$, for example, Eq. 19 yields

$$(1 - 0)(\mathbf{V}_7 - \mathbf{V}_3)\mathbf{Y}_{37} - (1 - 0)(-\mathbf{I}_{74}) = 0,$$

where each branch is counted only *once*, rather than twice as called for by the full summation. For $\mathbf{V}_3$ and $\mathbf{V}_1$, respectively, we have

$$(1 - 0)(\mathbf{V}_3 - \mathbf{V}_7)\mathbf{Y}_{73} + (\tfrac{1}{2} - 0)\frac{\mathbf{V}_3 + \mathbf{V}_2}{2}\mathbf{Y}_{46} = 0$$

$$(1 - 0)\mathbf{V}_1\mathbf{Y}_{41} + (\tfrac{1}{2} - 0)\frac{\mathbf{V}_1 + \mathbf{V}_2}{2}\mathbf{Y}_{45} = 0.$$

These yield

$$\mathbf{V}_1 = -V/5$$
$$\mathbf{V}_3 = V(7 - 8j)$$
$$\mathbf{V}_7 = V(9 - 10j).$$

From these all node voltages are readily evaluated, and are seen to agree with the results in Fig. 2.

Current $\mathbf{I}_2$ cannot be evaluated by a conventional current summation without first determining the transformer currents. This can be avoided by use of Eq. 20, where, as before, each branch is counted only once. This yields

$$\mathbf{I}_2 = (\tfrac{1}{2} - 0)\frac{\mathbf{V}_3 + \mathbf{V}_2}{2}\mathbf{Y}_{46} + (\tfrac{1}{2} - 0)\frac{\mathbf{V}_1 + \mathbf{V}_2}{2}\mathbf{Y}_{45} = \frac{I}{10}(1 - 21j),$$

which agrees with the previous case. Had node 2 been grounded, we could have determined $\mathbf{I}_2$ by inserting a fictitious voltage source between it and ground, re-writing the node voltages to account for the new voltage, using Eq. 20, and then setting the new voltage to zero.

The transformer currents can be calculated by conventional current summations, if desired. These lead to the values of branch current indicated in Fig. 3, which agree with the values obtained in the previous case.

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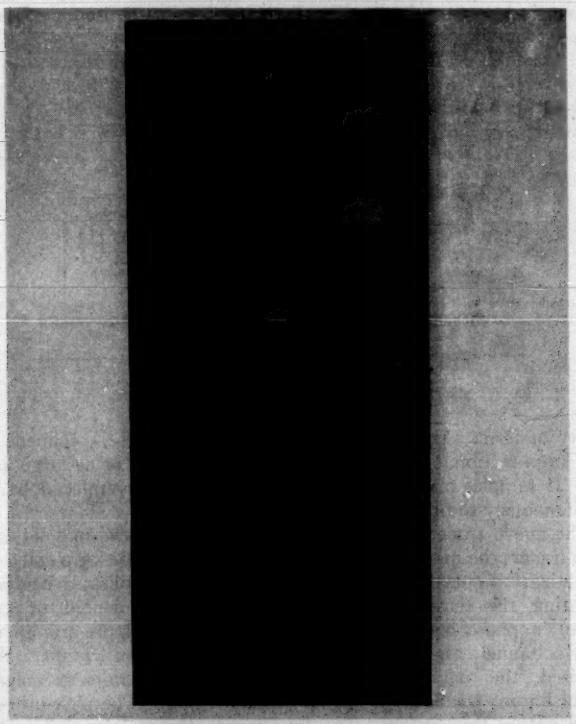
NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

A B–H CURVE TRACER

The magnetic properties of a metallic specimen are most commonly represented by a hysteresis loop, or B-H curve. A typical graph of B (flux density) *versus* H (field strength) can be interpreted in terms of the magnetic history of the specimen as well as its current properties. Because of the information it contains the hysteresis loop is valuable to

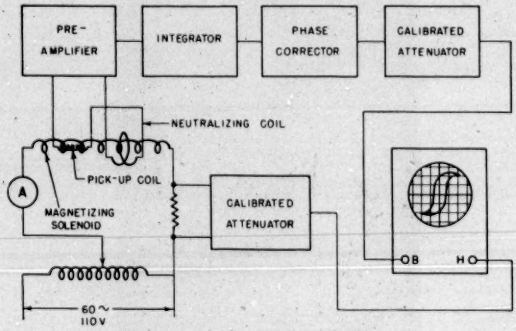


The B-H magnetization curve tracer. Not shown: the magnetizing solenoid housed in a separate, portable case.

those whose job it is to evaluate ferromagnetic materials for certain specifications or to develop new or improved magnetic materials.

Plotting a smooth and accurate B-H curve, however, requires the taking of many data which, without special equipment, is a prolonged and painstaking operation. The Electrical Engineering Division of the Laboratories has developed a device known as the B-H Magnetization Curve Tracer, which performs this operation automatically and rapidly. Two of these curve tracers have been built and are currently being used in production control laboratories of two magnetic-recording-wire companies.

Rather than recording discrete values of the variables, this device portrays the curve itself on a calibrated screen of a cathode-ray oscilloscope. The curve can then be recorded permanently in a photograph. Hysteresis loops can be readily displayed for magnetic materials in a



Cathode-ray B-H curve tracer.

variety of forms—powder, tape, wire, or rods. These specimens may have cross-sectional areas as small as 8×10^{-5} cm², or as large as 0.3 cm². If samples are smaller than 8×10^{-5} cm², several can be used simultaneously to obtain valid curves.

The curve tracer has three major components: the magnetization curve tracer; the magnetizing solenoid unit; and the pick-up coil. The magnetization curve tracer contains the power supplies; a bridge for indicating the temperature of the magnetizing solenoid (a safety device); a phase corrector, integrator network, and pre-amplifier for the B channel; the oscilloscope; and a calibration circuit. When energized, the magnetizing solenoid produces a uniform magnetic field of known strength within its central region. The pick-up coil is placed within this region. The flux linking the pick-up coil prior to inserting the sample is neutralized by auxiliary coils which can be

shifted back and forth within the solenoid housing. Two pick-up coils are supplied with the unit. One (with 35,000 turns) is suitable for measuring the properties of magnetic recording wire, tape, or powder samples. As many samples of small cross-sectional area and low permeability may be inserted into this coil as are necessary to obtain a loop of satisfactory size. The other pick-up coil (with 35 turns) has a longer bore than the first, and is suitable for measuring specimens whose cross-sectional areas are relatively large.

When a specimen is inserted into the bore of the pick-up coil, and when the magnetizing coil is energized, there is induced in the pick-up coil a voltage that is proportional to the time rate of change of B_s, the flux density in the specimen under test. This voltage, with its phase and amplitude adjusted, is applied to the vertical deflection plates of the oscilloscope. The energizing voltage across the magnetizing solenoid is applied to the horizontal deflection plates. By choosing proper amplification factors, the operator can obtain an accurate B-H curve on the oscilloscope screen. A calibrated template is used when reading or photographing the curve, which, together with the calibration circuit in the unit, makes the curve direct reading in B (gauss) and H (oersteds).

B. B. YOUNG

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

OIL FLOW IN PLAIN JOURNAL BEARINGS

A simplified method of analysis for the rate of oil flow in plain journal bearings has been developed by S. A. McKee, head of the National Bureau of Standards engines and lubrication laboratory. This problem is important to bearing design and operation; an adequate oil flow is essential to insure not only the maintenance of a load-carrying oil film, but also the disposal of frictional heat developed in the bearing.

The new method,[1] developed from data obtained on the NBS four-bearing friction machine in conjunction with the NBS-NACA lubrication research program, indicates that flow of oil from the ends of plain journal bearings fed through an oil hole, an axial groove on the unloaded side, or a circumferential groove may be affected by two factors. One results from the oil-feed pressure, and the other from the hydrodynamic-film pressure supporting the load. In the case of a bearing with a circumferential groove, the flow due to the hydrodynamic-film pressure is negligible in comparison with that due to the oil-feed pressure. For the other two types of bearings, however, both causes produce appreciable flow.

In the present work, analytical simplicity is achieved by postulating that the rate of oil flow, Q, is made up of two components, as follows:

$$Q = Q' + Q'' \tag{1}$$

where Q' is the flow rate caused by the oil-feed pressure and Q'' is the flow rate produced by the hydrodynamic-film pressure supporting the load. It is further recognized that, for any given type of oil-feed design, the two oil-flow components are related to bearing dimensions and operating variables. This relationship may be expressed by the following dimensionless equations:

$$\frac{Q'}{p}\frac{\mu}{C^3} = \psi_1\left(\frac{D}{C}, \frac{C}{L}, \frac{\mu N}{P}\right) \tag{2}$$

$$\frac{Q''}{P}\frac{\mu}{C^3} = \psi_2\left(\frac{D}{C}, \frac{C}{L}, \frac{\mu N}{P}\right) \tag{3}$$

where μ = absolute viscosity of the lubricant at atmospheric pressure and bearing temperature, p = oil-feed pressure, P = load per unit

* Communicated by the Director.

[1] For further technical details see, "Oil Flow in Plain Journal Bearings," by S. A. McKee, Paper No. 51-A-34, presented at ASME Annual Meeting, Atlantic City, N. J., November 25–30, 1951, and scheduled for publication by ASME.

projected bearing area, N = speed of journal, D = journal diameter, L = bearing length, and C = diametral bearing clearance.

As indicated above, the functions ψ_1 and ψ_2 depend upon the oil-feed design of the bearing. These have been determined experimentally for three basic designs: (1) one oil hole on the unloaded side of the bearing, (2) an axial groove on the unloaded side of the bearing, and (3) a circumferential groove. The results may be shown graphically by plotting the parameter containing Q'' against the Sommerfeld number, $S = (\mu N/P)\,(D/C)^2$, and by plotting the parameter containing Q' against the reciprocal of the Sommerfeld number.

In obtaining the above information, it is necessary to separate the observed total oil flow, Q, into its components Q' and Q''. For this purpose, a simple method was used. Briefly, the separation is done by plotting $Q\mu$ against p for constant values of $\mu N/P$. On such a plot, the data for each value of $\mu N/P$ fall on a straight line, the slope of which is equal to $Q'\mu/p$, while the intercept at $p = 0$ equals $Q''\mu$.

It is of interest to note that in the case of a bearing with a circumferential groove, the oil flow due to hydrodynamic-film pressure is negligible. However, for the other two oil-feed arrangements, this component is quite significant.

The good correlation of the parameter $Q''\mu/PC^3$ with Sommerfeld number, at different bearing clearances, is noteworthy. The differences obtained with bearings of different clearances when plotting $Q'\mu/pC^3$ against the reciprocal of Sommerfeld number are attributed to bearing out-of-roundness caused chiefly by deformations under load.

NEW ALUMINUM-PLATING PROCESS

A practical process for electrodepositing aluminum at room temperature has recently been developed by D. E. Couch and Abner Brenner of the National Bureau of Standards. Dense, ductile deposits of the metal are being obtained at the Bureau from a new type of organic plating bath[1] consisting of an ether solution of aluminum chloride and a metal hydride. The new bath is expected to find important application for electroforming articles with close inside tolerances, such as waveguides, and for providing various types of equipment with a thin protective coating of aluminum.

Because aluminum is so far above hydrogen in the electromotive series of the elements, it has never been deposited from aqueous solutions which always contain some hydrogen in ionized form. Aluminum is usually produced commercially from a bath of fused cryolite and aluminum oxide. However, this process must be carried out at a high

[1] For further details, see "A Hydride Bath for the Electrodeposition of Aluminum," by Dwight E. Couch and Abner Brenner, *J. Electrochem. Soc.*, Vol. 99, p. 234 (June, 1952).

temperature, and the metal is obtained in a fused state, unsuitable for electroplating or electroforming. Electrodeposition from nonaqueous solutions had been tried in the past, with some success, but the procedure was too difficult for practical applications, and the deposits were lacking in purity, ductility, and other desirable qualities.

The National Bureau of Standards is now conducting extensive research in an effort to develop methods for depositing in good physical form unusual metals such as molybdenum, tungsten, titanium, and zirconium. Both fused salts and baths of organic solvents are being investigated. One of the first metals studied was aluminum. The success of the aluminum plating bath, developed in the course of this work, may be expected to give added impetus to the current widespread efforts to obtain other metals in pure form from nonaqueous solutions.

The aluminum plating bath is prepared at NBS by adding either lithium hydride or lithium aluminum hydride to an ethyl ether solution of anhydrous aluminum chloride. For best results, the ether should be anhydrous and alcohol-free. The concentration of the aluminum chloride is not critical and may vary from 1 to 4 molar. Current densities may be as high as 4 or 5 amperes per square decimeter. However, if thick deposits are desired, the current density should not be greater than 2 amp/dm^2. Because of the high concentration of aluminum chloride in the ether, the bath is not as flammable as would normally be expected.

To prevent the entrance of moisture, the bath is prepared and used in a closed container consisting of a glass plating vessel with a tightly fixed polyethylene lid. Anodes of aluminum rod pass through the lid, and the objects to be plated (cathodes) are introduced and removed through a central hole which is ordinarily kept closed with a rubber stopper. If hermetically sealed, the bath will keep for several weeks; however, under ordinary operating conditions the solution slowly deteriorates and eventually gives streaked and brittle deposits.

No agitation of the bath is necessary. In fact, a quiescent bath is actually an advantage as the sediment from the anodes settles to the bottom of the vessel, making bagging of the anodes and filtration of the solution unnecessary. The composition of the bath is easily controlled since the only constituent that changes appreciably in concentration during the plating process is the lithium hydride. Occasional additions of lithium hydride increase the life of the bath, but in time the lithium hydride becomes insoluble and the bath can no longer be used.

When sufficient lithium hydride is used, the deposits are white, mat and quite ductile; but if the lithium hydride content is too low (less than 3 or 4 g/l), the deposits become hard, brittle, and gray. Still further reduction of the hydride content produces deposits that are dark and stressed and that often crack or peel from the cathode. Ordinarily deposits 0.01 in. or more thick are visibly crystalline, but this effect

can be reduced somewhat by adding a small amount of β,β'-dichloroethyl ether. Pitting, which frequently occurs in aqueous baths, is practically nonexistent in the ether bath.

Cathode and anode efficiencies for the process are approximately 100 per cent. Deposits 0.05 in. thick have been prepared at the Bureau, and thicker deposits should be possible if the sharp edges of the cathode are shielded to prevent treeing.

PENETRATION OF GAMMA-RAYS

Accompanying the increased use of radioactive materials in industrial research and medicine is a growing concern for the harm radiations from these materials can inflict on personnel. To aid in reducing these dangers to a minimum, the National Bureau of Standards, in cooperation with the Office of Naval Research and Atomic Energy Commission, has been conducting a theoretical and experimental investigation of radiation shielding devices. This particular program is devoted largely to studies of the distance radiations travel from their sources and the transformations they undergo during the process. The experimental phases of the work are designed to yield information that is essentially general in nature but which can be readily adapted to specific problems. Whenever possible, the experiments are carried out on a small scale or on a model. In this way, a wide variety of investigations can be made of the protection problems associated with devices such as very large nuclear reactors. Ultimately, the accumulated data will form a basis from which radiation barriers of the correct thickness and material may be designed for economical and safe protection.

One series of experiments on the penetration characteristics of gamma-rays utilized a large tank of water to simulate effectively an infinite medium. Radioactive cobalt (Co^{60}) served as the source of gamma rays, and two experiments were conducted, one to determine the intensity, the other to determine the quality of the radiation at varying distances from the source. In the intensity experiment,[1] conducted by Gladys White of the NBS staff, the Co^{60} source was suspended in the middle of a 60-ft. water tank. The amount of energy dissipated in the medium was measured with an ionization chamber and a Geiger-Müller counter at increasing distances from the source. The data showed that the decrease in the total intensity of the radiations closely approached the trend predicted in theories previously developed by NBS.

Determination of the quality of the radiations was delayed for some time because of a lack of techniques for measuring the energy

[1] Gladys R. White, "The Penetration and Diffusion of Co^{60} Gamma Rays in Water Using Spherical Geometry," *Phys. Rev.*, Vol. 80, p. 154 (1950).

spectrum of secondary electrons. Recently, however, a new method, that of scintillation spectroscopy, has been adapted by Dr. Evans Hayward, also of the NBS staff, to determine the energy of fast moving electrons generated by gamma rays. This technique was employed at NBS to observe the quality of the radiations by measuring the scintillations they produced in an anthracene crystal. These observations, too, agreed very well with NBS theoretical predictions.

As gamma radiation penetrates matter, it interacts with the substance either through the photoelectric effect, Compton scattering, or pair production. The predominance of one effect over the others depends on the energy of the quanta and the atomic number of the penetrated material. The characteristics of the radiation can be predicted accurately only under conditions corresponding to an infinite space or volume, and measurements must be made with detectors that do not appreciably disturb any of the quantities being observed. In the NBS experiment to determine radiation quality, a large tank ($12' \times 12' \times 25'$) of water was used that satisfactorily simulated the conditions of an infinite medium. In addition, the atomic number and density of anthracene crystals are so close to those of water, that to the electrons as they are scattered in the crystal, there appears to be no transition from water to the crystal. Hence, the pulse-height distribution representing the energy lost in an anthracene crystal would correspond to the energy distribution of the electrons produced in an equal mass of water occupying the same position.

The Co^{60} source and a counting instrument were suspended near the center of the water tank from a wooden rack. The counter consisted of the anthracene crystal, a Lucite light pipe, a photomultiplier tube, and a preamplifier. The whole unit was enclosed in an aluminum and Lucite container to protect it from the surrounding water. The Lucite light pipe separated the crystal from the photomultiplier, and a thin aluminum foil wrapped around the Lucite served as a light reflector. Thus, as the electrons are produced in the anthracene crystal, the light they produce is guided down the Lucite tube and the scintillations are detected by the photomultiplier.

The counter was designed to measure the energy of only those electrons produced in and totally absorbed by the anthracene crystal. Tests were made to ascertain that only these electrons were detected. A negligible contribution to the measured energy distribution was made by all those electrons that were created outside but scattered into the crystal and those created inside and scattered outside the crystal. Additional checks were run to determine the sensitivity of the spectrum to the length of the Lucite light tube and to the presence of a second photo-multiplier tube close to the crystal. In all instances, the effects were negligible.

The resolution of the counter was determined from the width of the internal conversion line of Cs^{137} (a source of a monochromatic radiation). A very weak Cs^{137} source was prepared, covered with a formvar film, and placed directly in contact with the crystal so that the beta spectrum and the electrons arising from the conversion of the photons within the atom could be observed. The pulse height distribution resulting from the internal conversion line proved to be gaussian with a half width of 110 kev.

Six Co^{60} sources, ranging from 10^6 to 10^{11} disintegrations per second, were used at varying separations between the source and the detector. Different sources were used at each separation in order to have sufficiently low counting rates and thus avoid residual and accumulated effects from the very large number of low-energy electrons characteristic of these spectra. The relative intensities of the sources had been measured in pairs and compared to a NBS cobalt standard, by which specific activities are known within $\pm$ 6 per cent.

The electron spectra produced by 1.17- and 1.33-Mev Co^{60} gamma rays were measured at distances of 10, 20, 41.6, 81.6, 121.6, and 161.6 cm. from the source. In plotting the data, the ordinate scale was in terms of the number of electrons per Mev interval, per disintegration of the source, per gram of anthracene, and the abscissa scale was in Mev. In order to express the data for the ordinate in these units, it was necessary to know not only the counting rate but also the disintegration rate of the source and the window width of the counting instrument. The error in measuring the absolute intensities was $\pm$ 10 per cent, due mostly to uncertainties in the determination of the latter two quantities.

The energy scale for the abscissa was obtained by making a linear plot of the data and taking the two points of inflection at the end of the spectrum as corresponding to 0.96 and 1.12 Mev, the maximum energy Compton electrons that can result from the 1.17- and 1.33-Mev gamma rays. This calibration was accurate to about $\pm$ 1.5 per cent. The standard deviations based on the number of counts were all less than $\pm$ 5 per cent and almost all less than $\pm$ 3 per cent. The background was negligible because the large mass of water provided an excellent shield for stray radiation.

At ten centimeters from the source, the high-energy end of the spectrum is quite intense and two peaks, corresponding to the two gamma rays, are easily recognized. At greater distances the energy loss becomes more and more concentrated in the lower-energy electrons, and the rate at which the spectrum changes becomes lessened. At distances of a meter or more, the changes in the shape of the spectrum of these medium and hard radiations are too small to be measured with this low-resolution spectrometer.

LUNAR REFLECTION OF UHF COMMUNICATIONS

A radio message was transmitted for the first time by lunar reflection during a recent cooperative experiment conducted by the National Bureau of Standards and the Collins Radio Company. On November 8, 1951, ultra-high-frequency signals that had been reflected from the moon were received by the NBS field station at Sterling, Virginia, after having been transmitted from Cedar Rapids, Iowa. Those participating in the experiment were P. G. Sulzer, G. F. Montgomery, and Ross Bateman of the Bureau's Central Radio Propagation Laboratory and I. H. Gerks of the Collins Radio Company.

Radio waves had been reflected from the moon before, but they were usually received at or near the point of origin. In the NBS-Collins Radio experiment, on the other hand, the signals were transmitted so as to be received at a site 775 miles from the transmitter after reflection from the moon. The operating frequency was 418 megacycles, generated by a 20 kilowatt transmitter. Because the transmitting antenna in Cedar Rapids was a fixed structure, lunar reflection could be accomplished only while the disk of the moon was in the beam of radio energy (a period of approximately one-half hour). The antenna at Sterling could be rotated and turned in the direction of maximum signal strength.

Reflection of the signals apparently began as soon as the leading edge of the lunar disk entered the radio beam. The receiving antenna was rotated until the maximum signal strength was obtained—in a position pointing directly toward the moon. As the moon continued to move across the radio beam, the received signal strength increased. About 10 minutes after the initial contact, the signal strength reached its highest value. The operators in Cedar Rapids then hand-keyed the signal in Morse Code and transmitted the historic message "What hath God wrought!" The intensity remained at this maximum level for another ten minutes and then began to decrease as the moon passed out of the radio beam. The greatest signal strength received was about one-millionth as strong as the signal received by most commercial television receivers.

To verify the fact that the signal was reflected by the moon, the actual transmission delay was compared with the theoretical value. This value was determined from the geometry of the experiment—the relative positions of the transmitter, the moon, and the receiver—and the known speed of radio waves. The 2.5-second time interval that was measured agreed approximately with the theory.

The NBS-Collins Radio experiment thus provides additional information confirming the possibility that the moon can be used as a reflector for short-wave radio transmission during those times it would be in the proper position for reflection. Use of the moon as a reflector would have the advantage that the transmissions would be free from

interruption. In present long-range communications, the ionosphere is used as a reflector. This layer of electrically charged air, 100 to 350 kilometers above the earth's surface, sometimes undergoes radical changes during "ionospheric storms." When this happens, the long-range communication is partially or completely interrupted. If UHF were used, however, employing the moon as a reflector, these radio storms would have little effect on radio transmissions. Moreover, the moon would not have to be optically visible to act as a reflector, as clouds or other meteorological conditions do not affect the radio beam seriously.

As a result of these experiments, it is believed that a dependable radio system may be possible if the transmitter and receiver were engineered for the specific purpose.

THE FRANKLIN INSTITUTE

ANNUAL REPORT OF THE BOARD OF MANAGERS FOR THE YEAR 1951

The Annual Report of the Board of Managers for the calendar year 1951 is presented to the Institute's membership by the President on behalf of the Board.

Officers and Board of Managers

Richard T. Nalle was elected President at the annual Organization Meeting of the Board of Managers, serving for the fifth consecutive year in that capacity. The other officers elected for 1951 were: Executive Vice-President—Henry B. Allen; Vice-Presidents—Richard W. Lloyd, G. H. Clamer, Edward G. Budd, Jr., and James H. Robins; Secretary—Henry B. Allen; Treasurer—Hamilton V. Bail; Assistant Secretaries—Hamilton V. Bail and John Frazer; Assistant Treasurer—Richard W. Lloyd.

On April 18, 1951, the Board of Managers regretfully accepted the resignation of Horace P. Liversidge, who had served as a member of the Board since December 1931. Henry B. Bryans (then President of Philadelphia Electric Company) was elected on June 20th to fill the vacancy thus caused.

Hostess Committee

The Hostess Committee has continued to earn our gratitude for its interest in and its effective contributions to the Institute's needs. Continuation of the grounds-improvement program was the Committee's chief project for 1951.

Journal of The Franklin Institute

The January 1951 issue of the JOURNAL OF THE FRANKLIN INSTITUTE commemorated its 125th anniversary with a symposium "Science and Tomorrow," in which fourteen eminent scientists discussed possible future developments in fourteen branches of science and technology. The issue received favorable comment throughout the country, with reviews in many United States and foreign technical journals and in the press.

Fifty-four papers were submitted for consideration, of which eleven were published and twenty-seven were added to the list awaiting publication. The board of reviewers continues to merit the gratitude of the Publications Committee for its painstaking examinations of manuscripts.

Substantial increases in printing and engraving costs made it expedient to raise the subscription price from $8.00 to $10.00 a year, effective with the September issue; and the JOURNAL was thus enabled to continue on the present basis of 112 pages per issue.

Library

On January 1, 1952, the Library contained 139,796 bound volumes, 50,065 pamphlets, and 6976 maps. Acquisitions in 1951 totalled 3139 bound volumes (986 by gift, 810 by purchase, and 1343 by binding), 511 pamphlets, and 840 maps by gift.

Patent specifications from United States, Great Britain and Switzerland were received regularly to keep our files up-to-date. Three thousand United States patent specifications, missing from our 1914 to 1919 collection, were secured and presented as a gift by the Philadelphia Patent Law Association.

Another noteworthy gift was the entire library of the National Association of Watch and Clock Collectors.

Several choice rare books were added to the James T. Morris Collection.

Through subscriptions and exchange relations with 476 societies and institutions, over a thousand periodicals are being received currently. New periodicals were added during 1951, and many war-time issues were also received.

Despite the fact that service to members and readers required much of the time of the Library staff of eight persons, a considerable amount of re-cataloguing was accomplished.

Regular Library hours were continued, and 7481 persons made use of the Library. Persons who could not use the Library personally received service through the mail, by photostat, and by telephone. Subjects in the field of chemistry were the most frequently consulted during the year; and as the Library's collection of books, periodicals and patents in this field is one of the best in the country, much help was given to members and readers.

Meetings

Attendance at stated meetings and at meetings held jointly with various technical societies averaged 237. Dinners preceding these meetings were attended by an average of 120 members and guests. The Edward G. Budd Lecture was given by Harold E. Stassen (President of the University of Pennsylvania), and the Charles Day Lecture was delivered by Henry B. Bryans (then President of Philadelphia Electric Company). All addresses were timely and well received.

Guy Marriner gave his 200th lecture-recital on November 11th and was presented with a Certificate of Appreciation by the Institute. This event was the high-light of these ever-popular lecture-recitals, and was attended by an enthusiastic capacity audience. The Guy Marriner lecture-recitals are now in their 15th season with no diminution of interest.

Not only has the number of scientific and technical organizations using our Lecture Hall and other meeting rooms increased, but recently instituted educational courses for our technical personnel, as well as special exhibits and demonstrations for the Museum, have kept our rooms in almost constant use.

Membership

During 1951 membership increased by 295, making a total membership at the end of the year of 6246.

Three "Science Is Fun" lecture demonstrations, sponsored by the Membership Department as a more informal demonstration to supplement regular Institute lectures, were held. They were "What about Atomic Radiation" by Richard A. Weiss of the Signal Corps, "The Nature of Things" by Roy K. Marshall, and the Westinghouse Electric Corporation's "Theatre of the Atom." At this last meeting, held in the fall, three men who have been members of the Institute for fifty consecutive years were honored at the dinner preceding the meeting and were presented with gold keys and gold membership cards. The gentlemen so honored were James M. Caird (Municipal Water Supply Engineer, Troy, New York), Gano Dunn (President of the J. G. White Engineering Corporation, New York, New York) and W. E. Moore (President of W. E. Moore & Company, Pittsburgh, Pennsylvania).

Informal buffet supper meetings for members to meet the staff of the Institute and to study special exhibits more closely were an innovation. It is believed that this activity will give our members a better understanding of the work of the Institute staff and of the exhibit material in the Museum. These are limited in attendance to fifty persons (Institute members and their families only). The first one to "Meet the Astronomers," held in November, was fully attended; and we received very favorable reactions from those who were there.

Museum and Planetarium

Attendance showed a substantial increase over 1950, the principal improvement falling in the later months. Comparative figures are:

	Paid	*Free*	*Total*
1950	162,958	67,814	230,772
1951	174,369	78,838	253,207

Attendance receipts increased $3,787.00 for the year.

Exhibits:

No changes of major importance in the arrangement of the Museum are to be reported, but plans for projected exhibits have been approved by sponsors and will be realized the summer

of 1952. These will result in impressive changes to existing exhibits, proving that industrial sponsors are appreciative of the opportunities for cooperating in the preparation of instructive displays.

In order to meet the demands of the lay public for additional information on the principles and applications of atomic energy, a temporary display (designed and arranged by the Atomic Energy Commission and sponsored by *The Philadelphia Inquirer*) was shown. This was supplemented by a series of vivid demonstrations in Westinghouse Electric Corporation's "Theatre of the Atom."

A notable exhibition of the regional entries in the Ford Company's Industrial Arts Award for the encouragement of craftsmanship was held in Franklin Hall in July. Along similar lines of promoting interest in science and the crafts among local students was the third annual Greater Philadelphia Science Fair, under the joint sponsorship of *The Philadelphia Inquirer* and the Institute. This attracted 238 exhibitors.

Another temporary exhibit which proved unusually attractive was the model HO gauge railroad furnished by the Baltimore and Ohio Railroad.

The seventh annual Philadelphia Book Clinic and Show was held in April. This illustrated several phases of the Graphic Arts as practiced in the city.

The Aetna Life Affiliated Companies presented to the Museum complete fire prevention demonstration equipment which has become a regular feature of the staff demonstrators' presentations to visitors.

An exhibit of unusual nature was the Econorama, prepared by Mr. James D. Mooney of New York, which shows what happens to the family budget as well as the inter-relationships among supply and demand and price.

Careful selection of material offered to the Museum continues to be exercised because of restricted display and storage space. The most noteworthy loan to be reported is that of a silver cream jug presented to Benjamin Franklin by his English friend Dr. John Fothergill. This has been added to the slowly growing collection of Frankliniana.

Among the gifts received were:

A United Nations Flag, from Mrs. Eugene P. Wigner, Princeton, New Jersey.

A collection of original patent models of printing equipment, from the Drake Press, Philadelphia, Pennsylvania.

A fragment of Benjamin Franklin's coat, from Miss Edith M. Bache, Philadelphia, Pennsylvania.

A Periometer for testing peripheral vision, from the Aetna Life Affiliates, Hartford, Connecticut.

Artificial gems, from the Kenya Gem Corporation, Philadelphia, Pennsylvania.

Decorative panels from an old railroad passenger coach, from J. R. Stulb, Philadelphia, Pennsylvania.

"Millionaire" calculating machine, from the Bethlehem Steel Company, Bethlehem, Pennsylvania.

Ship's steering wheel and standard, from Mr. R. C. Morse, Philadelphia, Pennsylvania.

Special Activities:

The Book Counter had a prosperous year with almost $12,000.00 gross sales.

The customary program of the Fels Planetarium, which had innovations each month, was enriched by two novel lecture-demonstrations on the weather and the 200-inch telescope. Special demonstrations for the benefit of local school children were given to crowded audiences.

Science Adventures for young people continued popular. Since these are limited to the summer vacation period, their success justified the organization of a Winter Workshop, which has attracted 21 students.

For the seventh consecutive year, members of the staff cooperated with WFIL-TV and public, parochial and private schools in the preparation and presentation of "Science Is Fun,"

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a television program transmitted over this commercial channel and directed principally to the students. Staff members also appeared on other television programs related to the work of the Museum.

Personnel

The Institute staff increased by 77 employees—from 409 employees on January 1st to 486 employees on December 31st. There is recorded with regret the death of Frank Evans on July 16th. Mr. Evans had served faithfully as a guard since October 1933.

By an amendment to the Social Security Act, enabling non-profit organizations to participate, the Institute elected coverage of its employees effective April 1st. The pension plan, as it had been established in 1950, was continued, following the expressed desire of all employees. Group insurance, initiated in 1949, was also continued.

Richard W. Lloyd, volunteer Director of the Personnel Department since its inception in 1942, resigned in November 1951 due to his inability to assume the full-time directorship necessitated by the present magnitude of Institute operations.

Plant Operations

Many changes in our building have been made in recent years to accommodate our fast-growing research activities. Equipment, now in use for 18 years, needs constant maintenance and repair; and one major overhauling and installation project was essential during 1951 due to the breakdown of our water-cooling system. Increased activities in the building and increased use of our facilities have made it imperative to procure the latest type floor-cleaning equipment for more efficient housekeeping with no additional personnel. Security measures necessitated increased watchman service in the main building during hours when offices and laboratories are closed.

Public Relations

During 1951 the Public Relations Department greatly expanded its uses of the various communications media (principally television—with over 100 appearances) to reach various publics and to inform them of the many activities and events taking place at the Institute. This, of course, resulted in a more general effectiveness in the work of this department. The continued support of the press, radio and television industries is greatly appreciated.

The *Institute News* was published ten times, two months being covered with a single issue twice in the summer months.

An increased number of speeches and personal appearances before civic and social organizations was effected.

Several special previews were held in connection with Planetarium demonstrations. A breakfast in Franklin Hall honoring Senator Duff; a dance in the Planetarium given by the Pennsylvania Hospital; a special Planetarium demonstration for The Pennsylvania Roadside Council; and a dinner in the Planetarium sponsored by the Philadelphia Chapter of American Institute of Architects were arranged by this department.

Promotion of special Museum exhibits, events, and Laboratories' activities (mentioned elsewhere in this report) were coordinated through the department. Preparation of advertisements, brochures, pamphlets, etc., were all 1951 promotional responsibilities.

Finally, Medal Day ceremonies and a survey made by the Charles Morris Price School required the attention of the department. In addition, several hundred inquiries regarding the Institute or Benjamin Franklin were answered by supplying pertinent information.

Committee on Science and the Arts

The Committee on Science and the Arts met nine times, and thirty sub-committee meetings were held. Twelve Medals and two Certificates of Merit were awarded at the annual Medal Day exercises in October. The Franklin Medal was awarded to Sir James Chadwick. Two George A. Hoadley Certificates, given for long service on the Committee, were presented during the year: One to Mr. William L. Brown, 3rd, for twenty-five years of service; and one to Dr. H. Jermain Creighton for thirty-five years of service.

Research Laboratories

The Bartol Research Foundation:

The Bartol Research Foundation continues to engage in work under the following heads:

1. Researches in nuclear physics and cosmic rays.
2. Researches in matters pertaining to thermionic work.
3. Researches on secondary electron emission from metals.
4. Completion of the construction of the new Van de Graaff generator for positive ions and for a voltage range between five and ten million volts.
5. Use of the small linear accelerator already completed, and the design and construction of a larger unit for subsequent work.

A Cockroft and Walton set for special investigations on neutrons has been constructed and brought into operation. Work on the development of an ion source for use in high voltage machines has been completed, and considerable success has been achieved as regards maximum current with minimum gas flow. The small generator has been removed from the old building. Its operating facilities have been improved by the substitution of Selsyn in place of manual control. The machine has been set up in the new building.

In the cosmic ray fields, experiments in Minnesota secured further important information concerning the primary cosmic ray intensity at high altitudes.

B-29 airplane flights with cosmic ray telescopes secured new important data concerned with the latitude variation of the cosmic radiation, and an investigation carried out on Hogback Mountain in Shenandoah National Park during the summer added important new information through our knowledge of the mean life of the mesotron.

Our work on scintillation counters has been attended with considerable success; and this tool is becoming of increasing importance in various phases of our laboratory's work.

Under contract with the Frankford Arsenal, the laboratory is training an employee of the Arsenal in radioactive techniques, using as a basis for such training an investigation having to do with the diffusion of atoms in solids.

Our work on thermionics, semi-conductors, and the reflection of electrons from metals has continued with success throughout the year.

The educational program, inaugurated in harmony with a similar program of The Franklin Institute Laboratories for Research and Development, is being operated with success and enthusiasm on the part of the participants.

In addition to guiding the work of the laboratory, the Director also functioned as Chief of the Radiological Branch of the Technical Division of the Philadelphia District Civil Defense Council, with Dr. C. E. Mandeville as Deputy.

The shop and glass-blowing facilities of the laboratory have continued to function effectively and to the advantage of the work of the Foundation.

The laboratory as a whole is operating under six Government contracts, and its potentialities are further enhanced by its own budget.

The Biochemical Research Foundation:

The year 1951 was marked by effort under the difficulty of obtaining mature investigators to replace those lost to Government positions and to industry. Fortunately, enough of a central core of qualified workers remained with us to keep our research up to its previous quality. Physicists, organic chemists, enzymologists are needed, but in general they are in short supply due to the international situation which has enlarged the Government agencies as well as the demands of industry.

The character of biochemical research is becoming altered, and this demands new approaches. It is now more enzymatic and thus related to organic chemistry explanations. This is most marked in cancer research, which is always the chief effort of the Foundation. New phases of this area of research have been opened up, and interesting studies have been published which, it is hoped, will give cancer research in general a new approach. The general emphasis is upon cytochemistry and an effort to explain the intimate processes of cell metab-

olism. This has opened a new field for the Foundation, which—with its departments of physics, chemistry and cytology—is peculiarly adapted for it.

The publications appearing from the Foundation's laboratories have been many and well received. The eleventh annual volume of research publications, which have been in great demand, will shortly appear.

The Franklin Institute Laboratories for Research and Development:

The past year was a year of growth in terms of sponsored research handled as well as in terms of personnel and research facilities. New programs were instituted for procuring technical personnel, for advanced education and for training staff members in diversified fields in an effort to make better use of their talents, and for including personnel in important advisory participation with management. Several noteworthy sponsored research projects were completed in 1951; many were continued; and many new projects were begun.

The amount of research handled in 1951 was larger than in any previous year; the dollar volume of Laboratories' projects was $2,121,000.00, exceeding the 1950 mark by $552,000.00. Although most of this volume is attributable to needs by our Government, $212,000.00 was derived from industrial contracts.

Personnel increased correspondingly, there being 274 persons employed at the end of the year compared with 211 at the beginning. Of the 274, 221 were employed in technical capacities.

With this increase in activities, the existing 57,000 square feet of space available to the Laboratories is taxed to the limit of efficient operation. At the end of the year, plans were being formulated to provide additional space for 1952.

In an effort to offset increasing difficulties in procuring qualified technical people, steps were taken to improve the personnel procurement program. Universities and colleges throughout the Eastern area were canvassed by members of the staff who interviewed promising near-graduates and recommended the best for hiring. The success of these visits in 1951 prompts the Laboratories to continue them in 1952.

Supplementing this personnel procurement program, and of equal importance, measures were taken to increase the usefulness of those already employed. In July, a Lecture Planning Committee was formed to plan a series of lectures and seminars which would benefit not only individuals but the Laboratories as well by broadening the experience, hence the versatility, of researchers in every section. Very soon after the formation of the Committee, the lectures were begun, with qualified staff members or guest lecturers speaking on subjects that ranged in diversity from glass-blowing and report writing to statistical analysis and problems in structural engineering. In addition, several local universities were persuaded to include in their curricula certain courses designed to meet the needs of Laboratories' staff members.

As a result of the Laboratories' policy of encouraging staff members to publish technical papers, many were written under the auspices of the Laboratories and were published in authoritative journals or presented before technical societies.

At the Exposition of the American Association for the Advancement of Science held in Philadelphia near the close of the year, the Laboratories displayed and demonstrated some interesting laboratory apparatus. The demonstrations and available literature drew very satisfactory attention, and further strategic use of the display booth is planned.

Industrial sponsors in 1951 were: Academy of Natural Sciences; Allied Chemical and Dye Corporation; American Liberty Oil Company; Armstrong Cork Company; Atlantic Refining Company; Bethlehem Steel Company; Bird and Son, Incorporated; The Budd Company; Burrough Adding Machine Company; Certain-teed Products Corporation; Cities Service Research and Development Co.; Congoleum-Nairn, Incorporated; Doehler-Jarvis Corporation; The E. I. du Pont de Nemours & Co., Inc.; Electric Storage Battery Company; Flintkote Company; Foote Mineral Company; General Electric Company; Globe Oil & Refining Company; Gulf Research & Development Company; C. B. Hill Company; J. R. Hollingsworth Corporation; Hood Rubber Company; Houdry Process Corporation; The W. K. Kellogg

Foundation; Lee Plastics Company; The Lehon Company; Link-Belt Company; Morganite, Incorporated; Ohio Oil Company; Owens-Corning Fiberglass Corporation; Pabco Products, Incorporated; Pennsylvania Salt Manufacturing Co.; Redevelopment Authority of the City of Philadelphia; Philadelphia Electric Company; Pittsburgh Corning Corporation; Presto Recording Corporation; Proctor and Schwartz Company; Radio Corporation of America; Raytheon Manufacturing Company; The Richardson Company; Richfield Oil Corporation; Rohm & Haas Company; Selas Corporation of America; Sharples Corporation; Sinclair Research Laboratories, Inc.; Standard Oil Company; Stauffer Chemical Company; Stokes Molded Products, Inc.; Sun Oil Company; Texas Company; United States Rubber Company; Westinghouse Electric Corporation; Witco Chemical Company; Vibradamp Corporation; and H. L. Yoh Company.

New Laboratories:

Applied Hydraulics: Building of the new Applied Hydraulics Laboratory was completed during the first half of the year, in time to take on work for both the Navy Department and the Westinghouse Electric Corporation. By the end of the year, the new laboratory was so crowded that further expansion seemed inevitable if we wished to carry on the work.

Solid State Physics: Plans were made to organize a Solid State Physics Group along the lines of the existing groups (Electrical, Mechanical Engineering, and Chemistry and Physics). In October the new group, under the direction of Dr. Foster C. Nix, moved into its laboratory. Four projects are presently delegated to this group: Magnetostriction, Ferromagnetism, Effects of Radiation on Solids, and Thermocouple Materials. Special equipment includes instrumentation for electrical and magnetic measurement, as well as furnaces for making metallurgical samples—an Ajax spark-gap induction furnace, a molybdenum-wound vacuum furnace and other small resistance furnaces. A highly competent staff of scientists is being assembled to carry on this work.

Projects Completed during 1951:

Projection Magnifier: Under sponsorship of the Kellogg Foundation, development of the projection magnifier as a reading aid to the semi-blind was completed; and plans were made to put the instrument into production. Several prospective manufacturers were approached, and one was finally chosen. Negotiations by the sponsor with this manufacturer were under way when the year closed.

National Asphalt Research Center: The first meeting of the sponsoring companies—producers and users of asphalt who wish to know more about the basic properties of that material—was held at The Franklin Institute on January 29, 1951, with 21 companies represented. Additional sponsors were drawn to the program as the year wore on until, by the end of the year, 27 companies were members. It is anticipated that all will renew their contracts, and an even larger membership is forecast for 1952.

Odor Abatement: The work performed for the City of Philadelphia on odor abatement was successfully completed during 1951 and received wide acclaim from City officials. Papers on this work were presented at the annual meeting of the American Association for the Advancement of Science in December; the response was very satisfactory. The new City administration has acknowledged that it is well plaesed and would recommend that City Council appropriate funds to carry on this work.

Air Traffic Control: Projects sponsored by the Air Navigation Development Board (CAA), which were completed in 1951, led to new work that continued where the old left off. The value of the simulatory methods developed by the Laboratories for evaluating air traffic control systems has been recognized by the sponsor, and the new work consists of applying these methods of air traffic control to particular air terminals where traffic is exceedingly heavy. The Laboratories maintain at various major air terminals a fleet of trucks containing equipment for recording and monitoring air communications, and gathering other information necessary to evaluation of traffic control procedures. Altogether, projects for the Civil Aeronautics Administration continued at a brisk pace and with continued success.

Projects Continued through 1951:

Projects begun prior to 1951 and continuing through the year, exclusive of the above-mentioned, were: Guidance Devices for the Blind, Friction and Ballistics, High Polymer Research, Radiation and Thermodynamics, Engineering Mechanics, Design of Industrial Equipment, Design of Ordnance Equipment, Electronic Computing Devices, Electrical Power Drives, Colloids.

Friends of Franklin

The group of public-spirited individuals and companies known as "Friends of Franklin" continued its generous support of the Institute, contributing $26,457.50 in 1951.

Finances

The financial position of the Institute at December 31, 1951, as compared with December 31, 1950, is reflected in the following summarized comparative statement adapted from the report of our auditors (Messrs. Lybrand, Ross Bros. & Montgomery), a complete copy of which is available to members for examination in the office of the Treasurer:

Assets	*Dec. 31, 1951*	*Dec. 31, 1950*
Operating Funds		
Cash	$ 146,021	$ 107,519
Amounts due on laboratory research contracts	375,191	389,858
Other receivables	47,737	31,560
Inventories	38,967	33,908
Deferred charges	17,065	21,143
Total operating funds	$ 624,981	$ 583,988
Endowment Funds		
Investments at book values (market values $5,262,379 and $4,635,096, respectively)	$4,120,854	$3,625,531
Uninvested principal cash	22,017	25,479
Advances to operating funds	51,000	75,000
Advances to plant and property funds	145,000	220,000
Income cash	71,247	58,780
Total endowment funds	$4,410,118	$4,004,790
Plant and Property Funds		
Cash	$ 784	$ 784
Building, equipment, fixtures (less reserves of depreciation of $846,649 and $669,661, respectively)	3,719,324	3,844,011
Advances to operating funds	366,218	316,531
Total plant and property funds	$4,086,326	$4,161,326
Total assets	$9,121,425	$8,750,104
Liabilities and Funds		
Operating Funds		
Accounts payable, accrued expenses and deferred income	$ 170,234	$ 129,030
Advances from endowment funds	51,000	75,000
Advances from plant and property funds	366,218	316,531
Capital stock	10,480	10,580
Operating funds balance	27,049	52,847
Total operating funds	$ 624,981	$ 583,988

Endowment Funds	*Dec. 31, 1951*	*Dec. 31, 1950*
General and restricted endowment funds and funds functioning as endowment including those of Bartol Research Foundation	$4,338,871	$3,946,010
Undistributed and unexpended income	71,247	58,780
Total endowment funds	$4,410,118	$4,004,790
Plant and Property Funds		
Contributed funds	$3,941,326	$3,941,326
Advances from endowment funds	$ 145,000	220,000
Total plant and property funds	$4,086,326	$4,161,326
Total liabilities and funds	$9,121,425	$8,750,104

Comparative results of operations for the years 1950 and 1951 are shown in the following summarized statement of income and expenses of Operating Funds, likewise adapted from the auditors' report:

Operating Revenue	*1951*	*1950*
Membership dues	$ 58,379	$ 56,130
Museum admissions and other museum income	112,223	101,016
Laboratory research projects	2,120,905	1,567,463
Other activities	46,436	45,068
	$2,337,943	$1,769,677
Operating Costs and Expenses		
Museum	$ 294,494	$ 265,036
Library, less amounts transferred to other departments	65,793	61,483
Laboratory research projects	2,012,074	1,492,975
Other activities	74,586	69,758
Administrative and general expenses, less amounts transferred to other departments	77,101	62,695
	$2,524,048	$1,951,947
Operating loss	$ 186,105	$ 182,270
Other Income		
Income from endowment funds	$ 89,614	$ 93,597
Gifts from Friends of Franklin	26,457	25,443
Appropriation, Commonwealth of Pennsylvania	40,746	38,500
Other income	$ 15,474	$ 16,903
	$ 172,291	$ 174,443
Deficit before other charges	$ 13,814	$ 7,827
	1951	*1950*
Other Charges		
Interest	$ 7,182	$ 9,388
Transfer to endowment funds	—	16,100
Other charges	4,902	14,390
	$ 12,084	$ 39,778
Excess of costs and expenses transferred to Operating Funds balance	$ 25,898	$ 47,605

The excess of costs and expenses shown above is after charges for depreciation and amortization of buildings and equipment amounting to $182,355 for 1951 and $180,843 for 1950.

Respectfully submitted,

By order of the Board of Managers,
S. WYMAN ROLPH,
President

MUSEUM

To celebrate the 500th anniversary of the birth of Leonardo da Vinci the Museum has arranged an exhibition of models made from drawings in his note-books and other material to illustrate the achievements of one of the greatest minds of all time.

Leonardo began the study of nature as the vehicle which would conduct him to mastery of artistic design. In his treatise on painting he discusses the various fields of practical science necessary for the artist's education. Then, as he went along, his studies of nature gradually liberate themselves from art and he records a whole series of observations dealing with unexplored natural phenomena which lay outside the confines of art. He became a versatile genius, an investigator with a passion for experimentation, whose ambition appears to have been to understand the whole universe. His goal was to grasp the forms and laws of nature and life as they were revealed to his penetrating vision and to use his superb artistic ability to set forth his observations in marvellous drawings, some of which permit the construction of models to realize his ideas.

The great wealth of his studies is spread over thousands of pages in his notebooks so that it conveys the impression that he had in mind an encyclopaedic demonstration of all man's knowledge of nature. The broad sweep of his mind is amazing. Practical mathematics (like geometry) and optics he seems to have accepted as the basis for his knowledge. Then follow mechanics as the science of physical forces in both organic and inorganic nature; biology as the science of growth in organic nature; cosmology as the study of inorganic forms and the forces at work within them.

The exhibition in the Museum is rich in specimens of his work in mechanics, and from them we perceive him to have been both theorist and inventor. Throughout his life he remained an indefatigible and inventive engineer, as the many sketches in his notebooks prove. The models in the exhibition are constructed from these original sketches and they demonstrate how practical Leonardo's designs are. His inventions are innumerable but many of them were not designed for their own sake but only to illustrate the application of certain mechanical principles. For example, he was much interested in the transmission of power and saw many opportunities for performing this by mechanical means and applying it to specific problems. He tested transmissions by means of gears and pulleys in various pieces of apparatus, showing a clear grasp of the principles of the inclined plane, the lever, and friction.

Leonardo recognized that he was an empiricist, but he passes from practice to theory in such manner that he strives to secure for the theory a wider application in practice. One perceives this in all his mechanical work but it is also emphatically displayed in his architectural studies. We see it also in his hydraulic projects which had their origin in his investigation of the properties and movement of water. All of his work reveals that Leonardo's scientific theories and methods were based upon observation, which was the direct result of his artistic nature. He offers the most remarkable example of a combination of the artist and the scientist in a single mind, which had the additional faculty of employing either side of his nature to aid and develop the other.

This exhibition of his work has been rendered possible by the cooperation of the International Business Machines Corporation and members are urged to make the effort to see it. It will be displayed from June 24th to July 27th.

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, April 9, 1952.)

The following report was presented for final action:

No. 3248: Clamer Medal.

This report recommended the award of the Francis J. Clamer Medal to Cyril Stanley Smith, of Chicago, Illinois, "In recognition of his work leading to knowledge of the basic factors of the metallurgical behavior of elemental plutonium essential to the development of nuclear energy."

(Abstract of Proceedings of Stated Meeting held Wednesday, June 11, 1952.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 11, 1952.

MR. HOWARD STOERTZ in the Chair.

The following report was presented for final action:

No. 3254: Brown Medal.

This report recommended the award of the Frank P. Brown Medal to Fred N. Severud, of New York, New York, "In consideration of his outstanding engineering accomplishments in the field of construction, characterized by intelligent thinking, whereby he has combined sound engineering principles with ingenious and novel designs to produce improvements giving simplicity, economy, usefulness and pleasing effects."

JOHN FRAZER,
Secretary to Committee.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS

June 18, 1952

ACTIVE FAMILY

Ralph V. Coles — Earl V. Edkins — R. M. Hibbs

ACTIVE

Alfred Bitterlin — W. Harold Edmunds — G. H. Newcomer
J. Lloyd Bohn, Ph.D. — William T. Gray, Ph.D. — Herbert G. Pascalar
John P. Carty — W. R. Huguenin — Arnold Raines
John H. Day — William H. Johnson, Jr. — Ellwood M. Schofield, Ph.D.
William W. Dickhart, III — Richard M. Kerwin — T. Kite Sharpless
Herbert S. Dordick — John I. Kirsch — J. H. Shreiner
Nicholas J. Doto — Samuel K. Lessey — L. Pierre Teillon
Harrison E. Dow — Mrs. Fred C. Newcombe — Alexander E. Troop
Benjamin L. Dyner — — Charles N. Vallette

ACTIVE NON-RESIDENT

Capt. Everett L. Gayhart — — Frederic F. Grant

NECROLOGY

J. Harold Austin, M.D. '49 — E. Stanley Bowers, Sr. '42 — Shippen Lewis '39
John Hampton Barnes '12 — R. A. Hoover '46 — George Woodward, M.D. '04
— Samuel F. Houston '47 —

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Wednesdays and Fridays from 9 A.M. until 5 P.M.; Thursdays from 2 P.M. until 10 P.M.; Saturdays from 9 A.M. until noon.

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BURRARD, GERALD. The Identification of Firearms and Forensic Ballistics. 1951.

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JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

SHAW, M. C. A Yield Criterion for Ductile Metals Based Upon Atomic Structure.

BURROWS, W. H. Some Properties of Hyperbolic Coordinate Systems.

TOMOVICH, RAJKO. A Universal Unit for the Electrical Differential Analyzer.

PORTER, W. C. AND W. E. RAMSEY. A Circuit for the Limitation of Discharge in G–M Counters.

MOON, PARRY AND DOMINA EBERLE SPENCER. Separability in a Class of Coordinate Systems.

POLNAUER, FREDERICK F. Biomechanics—A New Approach to Music Education.

NEWTON, GEORGE C., JR. Compensation of Feed-back Control Systems Subject to Saturation.

SILBERSTEIN, LUDWIK. Properties of an Exponential Size-Frequency Distribution of Finite Extent.

KRZYWOBLOCKI, M. Z. On the Invariants in the Turbulence in Compressible Viscous Fluids.

THE FRANKLIN INSTITUTE

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BOOK REVIEWS

THE DESIGN AND ANALYSIS OF EXPERIMENTS, by Oscar Kempthorne. 631 pages, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $8.50.

The recent awakening of many workers in the physical and applied sciences to the value of statistical methods has resulted in the publication of numerous books on this subject. These have varied from highly theoretical discussions to relatively simple explanations of the common methods and formulas to be applied in solving specific problems.

The present book is of the theoretical type, containing very few numerical examples and almost more mathematical equations than text. A knowledge of elementary statistics and the theory of errors is assumed, as well as of considerable mathematics, including matrix algebra. Unfortunately, the text is not written in a simple manner, and the meaning of even relatively simple discussions is sometimes obscure.

The first fifth of the book is essentially introductory, beginning with an interesting and penetrating discussion of the scientific method and how statistics fits in, followed by a brief review of elementary statistical techniques and experimental design, then by mathematical discussions of the method of least squares, multiple regression, analysis of variance, and of multiple classifications. The remaining four fifths consists of a logical development of the theory of experimental design: randomization, Latin squares, plot technique, experiments with several factors, confounding, split-plot experiments, fractional replication, lattice designs, and incomplete blocks.

This book is definitely not recommended to those desiring only a cursory knowledge of the subject or seeking only to find equations in which to substitute numbers to solve problems. It is hard reading and should not be attempted by anyone not fluent in the language of mathematics. But those who do struggle through the book will find themselves well rewarded, having gained a deeper insight into the whole subject as well as a basic knowledge of many techniques of experimental design. It is this insight and basic understanding that are so important in avoiding the pitfalls of misapplication so likely to occur if one merely puts numbers into formulas. Anyone who expects to spend a large portion of his time in the application of statistics to experimentation, unless in only a routine manner, should not fail to study this book.

NEWTON W. MCCREADY

ENGINEERING MATERIALS, by Joseph Marin. 491 pages, illustrations, 15 × 22 cm. New York, Prentice-Hall, Inc., 1952. Price, $8.70.

It is no longer enough to know only the physical properties of an engineering material in order to design a machine or structure. To be a competent designer, an engineer must be acquainted with all the criteria of design. Not only must he know such physical properties as yield and ultimate strengths, hardness, ductility and impact; he must be familiar with fatigue phenomena, stress concentration, creep characteristics, the properties of ferrous and non-ferrous alloys in order to choose the most suitable material; and he must be able to predict with some degree of certainty the expected loading conditions for his design. In short, he must be an engineer in the true professional sense of the word. In a broad, somewhat elementary fashion, Professor Marin of the Pennsylvania State College, discusses all these aspects of good engineering design in "Engineering Materials." For the student it is a sound introductory text, and for the practicing engineer, an excellent review of the criteria of design he may have forgotten.

Obviously written for the student, "Engineering Materials" is divided into three parts: Mechanical Properties, Specific Materials, and Materials Testing Machines and Strain Gages. Under Part One, static properties in tension and compression, shear and bending, and under combined stresses; fatigue, impact, and creep and temperature properties are discussed. Generally, each property is clearly explained including how it is used in design with problems to illustrate each application. In Part Two, the structure of materials and control of their

properties, ferrous and non-ferrous metals and alloys, and non-metallic materials are reviewed. Although brief and elementary, this section is nevertheless quite interesting and informative, and forms a good background for a deeper study of metallic and non-metallic structural materials. The final Part Three explains materials testing machines for simple and combined stress, for creep, for fatigue and for impact; and such strain gages as mechanical, optical and electrical are all treated. E. W. HAMMER, JR.

THE AURORAE, by L. Harang. The International Astrophysics Series, Vol. I, 166 pages, illustrations, 16 × 25 cm. New York, John Wiley & Sons, Inc., 1951. Price, $4.50.

Published as Volume I of the International Astrophysics Series this is one of the very few books written on a little understood phenomenon, the aurora. Of a highly authoritative nature, this technical book is intended primarily for the specialist in the field and the advanced student.

This reviewer's introduction to the aurora resulted from a solar observational program some fifteen years ago and the memory of searching the literature for information on the subject is still vivid. Aside from a popular book on the aurora published some five years ago there is nothing in book form of recent vintage available to the serious student. However it should be indicated that even though this book was published in 1951 it is already outdated by the recent developments by Gartlein and Meinel in this field. It is somewhat regrettable that the new information published in the *Astrophysical Journal* was not included. It is hoped that the next edition will use the wealth of material found in the publications of this country.

The book opens with a description of the auroral forms and height determinations. Here is found the introduction to the "nets" used by Stormer to facilitate height determinations. The range of heights of the various types and the effect of sunlight are exhaustively explored.

Included is a section on auroral sounds. The author, while not attempting to account for these sounds, gives numerous unimpeachable sources who have heard them. The evidence strongly indicates a correlation between faint whistling or crackling sounds and great aurora.

Chapter III deals with the spectrum of the aurorae. The instruments used for the analysis are described and tabulation of the auroral lines and bands and the identification of these are given. Included is a section on the temperatures of the upper atmosphere as determined by the nitrogen bands and while the results are as yet inconclusive it indicates an approach to the solution of an exceedingly interesting problem. It would be curious if these temperature determinations were substantiated by the results of the rocket explorations of the upper atmosphere.

There is a chapter which deals with the earth magnetic storm and aurora relationship. The author could have found very interesting correlations in the United States during the great magnetic storms of 1939 and 1940. Because this country is so highly industrialized the frequency of disturbances would be greater than elsewhere.

The remainder of the book deals with the highly technical corpuscular theory of the aurorae, the upper atmosphere and the auroral and finally the ionosphere.

"The Aurorae" fills a gap in our knowledge of this phenomena. I. M. LEVITT

PROPAGATION OF SHORT RADIO WAVES, edited by Donald E. Kerr. 728 pages, drawings and photographs, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $9.00.

Without doubt, this volume represents the most complete and up-to-date collection of data on the subject of microwave transmission. The treatment of the subject is both theoretical and empirical, taking into account a large body of measured data and the analytical predictions which directed the course of these measurements.

Considering the volume's contents generally, the principal concern of the authors has been the effects of the atmosphere and its refractive influence upon microwave propagation. The attenuation characteristics that result from specific meteorological phenomena are treated extensively. This work in some respects provides a foundation upon which several important radar system design factors may be evaluated. For example, angle-of-arrival of the radiation affects radar echoes and has a definite determining influence upon antenna design considerations. Propagation phenomena, of specific types treated, naturally must be taken into

account in predicting radar system powers and sensitivities required to produce any specified performance characteristics.

Although the subject of this book may be thought of as being somewhat specialized, it is timely and of extreme importance at this point in our development of high-information-content transmission and relay systems. The information, originally gathered for the purpose of improving our wartime radar systems, is fortunately available now that it is needed for general communications developments. The communications channels that will be used to the limits of their capacity in the future will of necessity rely on a better understanding of the propagation phase of the engineering problem as well as the components and other system improvements. The timeliness of this subject is further emphasized by the fact that the Franklin Institute Laboratories are currently engaged in a very large theoretical and experimental program concerned directly with microwave propagation studies.

This treatise has the advantages of multiple authorship. That is, the various sections are written by specialists with broad personal experience in their own fields. In all, there are thirteen contributing authors, each of whom is highly competent. The presentation consists primarily of new work done by these men at the MIT Radiation Laboratory and is not a review of work done by others. Although the book may not have a popular appeal to radio engineers in general practice, it is a very necessary volume of the Radiation Laboratory Series for complete coverage of radar and related techniques. C. W. HARGENS

INTRODUCTION TO ELECTRONIC CIRCUITS, by R. Feinberg. 163 pages, diagrams, 15 × 22 cm. New York, Longmans, Green and Co., Inc., 1952. Price, $3.50.

"Introduction to Electronic Circuits" is based on a lecture and laboratory course given by the author to honor students in Electrical Engineering and Physics, at the University of Manchester, and is intended for the undergraduate student of the subject. Numerous laboratory experiments are suggested to emphasize the theoretical and practical subjects covered in the text.

The book deals with external effects of electronic devices and their utilization in electronic-circuit technique. These effects are rectification, control of current, amplification, frequency multiplication and intermodulation, negative-resistance performance, and electronic-switch action. Electron-transit time effects are not considered.

The terms and symbols used are those of the British Standards No. 204 and 205, and differ slightly from those in common use in this country. Numerous illustrations, numerical problems, and references are contained within this abbreviated introduction to electronic circuits. A. D. HAY

ELEMENTS D'OPTIQUE ELECTRONIQUE, by Gaston DuPouy. Collection Armand Colin No. 267. 216 pages, illustrations, 11 × 16 cm. Paris, Librairie Armand Colin, 1952. Price, 260 fr.

According to popular legend the French mind is very logical, and this book may be taken as support for the legend. The arrangement of the material presented is extremely clear and logical. It is a small book whose purpose is to describe the physics of electron motion as it applies particularly to the electron microscope. By nature the book is essentially an elementary text book, so that the logical presentation is especially valuable. The French is simple, and the argument is clear and easy to follow. There are several places where it is possible to argue with some of the physics used, but in general these do not detract from its value as an exposition of the basic principles involved.

It is, however, in the arrangement of the subject matter, that the book is particularly pleasing. A general introduction sets forth in simplified form the laws governing the motion of electrons in static electric and magnetic fields. The theory and practice of electrostatic and magnetic electron lenses are then described in more detail in two sets of three chapters each. After both the principles and experimental arrangements have been thoroughly discussed, the defects and aberrations inherent in both types of lenses are described. A short chapter points out analogies between electron and optical lenses. Not until the final chapter is the actual subject of electron microscopes discussed. This arrangement gives the reader a chance to learn thoroughly the basic principles involved before being exposed to the many

details of the microscope itself. The chapter is completed by brief descriptions of the use of the microscope in various problems.

Where the language is not a handicap this book should be an excellent basic text of the principles used in the electron microscope, and should be quite valuable to those working with the instrument.

ALAN D. FRANKLIN

WATER: A STUDY OF ITS PROPERTIES, ITS CONSTITUTION, ITS CIRCULATION ON THE EARTH AND ITS UTILIZATION BY MAN, by Cyril S. Fox. 148 pages, illustrations, plates, 16 × 25 cm. New York, The Philosophical Library, 1952. Price, $8.75.

Written by Sir Cyril Fox, lately Director of the Geological Survey of India, this short book is the initial volume of a series of treatises on various aspects of water as supply, treatment for purification, sewage disposal, construction of water works and wells for the worthwhile purpose of fulfilling a rather critical need for such information. It is an interesting treatise, and since its aim of being as nontechnical as possible is largely realized, the layman should derive as much pleasure from it as the engineer does information.

Three parts comprise the volume: the natural history, the work done, and the utilization of water. Under natural history, the constitution, distribution and circulation of water are discussed; under work done, erosion, the action of underground water and deposition of sediments are described; and under utilization, general considerations of supply, water supply engineering and some concluding remarks on water rights complete this modest volume.

A number of interesting and rather fascinating facts are scattered throughout the book, although not necessarily pertinent to water. Perhaps the most amazing was the discovery, in 1900 in Siberia, of the frozen but complete carcass of a mammoth. Although extinct for several hundred years, the flesh of this animal was so well preserved that it was fed to the sled dogs. As the author emphasizes, this convincingly demonstrates the value of freezing in the preservation of meat. For years we have heard the term Artesian well. Aware, of course, that it refers to well water forced to the surface by its own pressure, how many know that the name derives from the province of Artois, Pas de Calais, France, where such wells were first discovered? In the Valley of Ten Thousand Smokes in Alaska, some fumes, escaping from vents, are sufficiently hot to fry bacon. During the volcanic explosion of Krakatoa, in the Sundra Straits, 50,000,000,000 tons of rock were scattered in a few seconds.

Do not assume from these few spectacular facts that the book is predominantly sensational. It is not; for considerable engineering data on water supply, rainfall, water storage, distribution and related aspects of water are presented. However, most of these data, since they are not generally widely disseminated, are interesting and, consequently, the book will be naturally popular.

E. W. HAMMER, JR.

THE THEORY OF ISOTOPE SEPARATION, by Karl Cohen. 165 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $2.00.

After a slow start consisting of two Forewords, two Prefaces, two Acknowledgments, but only one Introduction, this recent addition to the National Nuclear Energy Series takes us through a beautiful mathematical treatment of the practical separation of physically and chemically differing components, in this case, isotopes.

The book presents a completely mathematical treatment of the theory of "cascades" as applied to the separation of isotopes by the Manhattan Project. (Here a cascade may be defined as a series of repetitive and identical operations performed on a mixture of isotopes for the purpose of separation or concentration of the isotopes.)

The entire work was accomplished during the few years leading to the successful concentration of U-235 by the diffusion process and probably represents as concentrated an achievement of the application of fine mathematics to chemical engineering as any in the history of the modern design of chemical plants.

The first few chapters derive formulae applicable to ideal cascades and square cascades and determine cascade constants. An analysis is then made of the problem of control in plant operation; several specific methods of separation, not including the most important method of diffusion, are then treated mathematically.

This edition is solely of interest to those intimately connected with the problem of isotopic separation, to whom it is probably indispensable. On the human side, it is a tribute to the fine mathematical minds of the author and his colleagues. ALBERT L. MYERSON

TEXTBOOK OF ELECTROCHEMISTRY, Volume II, by G. Kortüm and J. O'M. Bockris. 882 pages, illustrations, 17 × 25 cm. Amsterdam, Elsevier Publishing Co., 1951. Price, $10.00.

This, the second volume of the revised and translated edition of Kortüm's original work, "Lehrbuch der Electrochemie," follows the same presentation as found in Volume I; that is, the authors present the fundamentals of electrochemistry without becoming involved in the many highly specific problems of the field. Thoroughgoing bibliographies found at the end of each chapter provide a ready source of up-to-date references for readers who wish more detailed information on particular problems.

The first four chapters in this volume are devoted to the presentation of the following phases of electrochemistry: (*a*) electrical phenomena at interfaces; (*b*) irreversible electrode processes such as concentration and activation overpotential, mechanical and chemical passivity, and corrosion of metals; (*c*) electrochemistry of gases including chemical reactions taking place under the influence of an electric discharge; and (*d*) experimental methods of electrochemistry. The final chapter consists entirely of tables of physico-chemical data such as physical constants of compounds, properties of electrolytes and electrolytic solutions, and electrode potentials. In the appendices are to be found problems arranged in order of increasing difficulty, with their answers, covering the material presented in each chapter of both volumes. These problems were included in order to familiarize the student with the many concepts of electrochemistry presented in this treatise. An alphabetically ordered subject index for the combined volumes is found at the end of this book.

The fourth chapter of this volume should be particularly useful to the research worker, since it outlines the most modern methods of electrochemical research techniques. This section is heavily annotated with references to the original literature sources which may be consulted by the research worker when the working details of some particular technique are required.

Taken together, these two volumes represent the most modern and comprehensive textbook of electrochemistry in the English language. This treatise is invaluable as both a text for students and a source book of information for research workers in the field.

D. H. RUSSELL

LET THERE BE BREAD, by Robert Brittain. 244 pages, 15 × 22 cm. New York, Simon and Schuster, Inc., 1952. Price, $3.00.

The opening sentences of this book inform us that 18,000 hungry mouths were brought into the world between the time we went to bed last night and this morning's breakfast-time. The author then goes on to compute that the population of this planet increases some twenty million yearly, and raises the question whether the resources of the earth can be trusted to support them. A student of Malthus could paint a very depressing picture of the chance of survival from this evidence, but Mr. Robert Brittain (who is both poet and specialist in the hopeful aspects of scientific research in promoting human welfare) takes a much more optimistic view. He knows, too, what he is talking about. The fact that the graph of most natural resources is going down while demand soars to unprecedented heights holds no terrors for him. Human ingenuity can restore the balance.

The source of Mr. Brittain's optimism resides in belief in man's creative genius to meet emergencies as they arise. Limiting his investigations to the production of animal and vegetable materials which are suitable for human consumption, the author of this thought provoking book systematically examines the various steps which must be undertaken to ensure adequate food supplies for the rapidly growing population on this plundered planet. Beginning with the problem of bringing more acres under cultivation and increasing the yield of acreage now in use, he tells of fascinating experiences which have resulted in ripening watermelons north of the Arctic Circle and of cool tree-shaded parks flourishing where, a few years ago, there was nothing but shifting sand dunes. He is quite emphatic in declaring that we have the scientific

knowledge and skill to make the deserts fruitful, and cites proof of the work that is being done around the Mediterranean. The Hylean Amazon and other projects are described to explain how tropical areas may be brought under the cultivation of productive field crops, and numerous examples of soil reclamation are quoted in detail.

The latter part of the book deals with the thorny problem of conservation, and Mr. Brittain has many interesting things to say upon it and its allied subject, water control. Here one sees how the author can be so optimistic as he blithely describes the great Colombo, Rhone, and other projects (largely based on T. V. A.) which are being put into practice all over the world. One chapter is devoted to the steps being taken to conquer the destructive agencies of uncontrolled water, wind, and disease. Having made exhaustive inquiries into the three main forces by which salvation may be gained—reclamation, conservation, and development, Mr. Brittain shows how mankind need not despair that the human race will end in the pit of Malthusian prophecy. The author does not overlook the difficulties, but these would be small indeed if those who have to solve them can be infected with his optimism and confidence. His book, written in simple non-technical language, is to be cordially recommended.

T. C.

PRINCIPLES OF GEOCHEMISTRY, by Brian Mason. 276 pages, diagrams, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1952. Price, $5.00.

The science of geochemistry is concerned with the chemistry of the earth and its various parts, the atmosphere, the hydrosphere, the biosphere and the lithosphere. Its purpose is to ascertain the quantitative composition of the earth and to discover the laws controlling the distribution of the elements. Although in existence as a separate science for over one hundred years, only within the past twenty-five years has it become prominent, particularly in the past decade under the stimulus of the rapid advancement in nuclear physics.

Since geochemistry is concerned with the structure of the earth, it is perforce concerned with the nature and origin of the universe and our solar system as well as of the earth itself. So the science not only appeals to the geologist and chemist, but to the astronomer as well. However, "Principles of Geochemistry" should interest anyone, dealing as it does with the nature of the earth and, although frequently technical, should be understandable to those with only a modest background in geology and the general sciences. It is an absorbing book.

Professor Mason, of the Geological Department of Indiana University, writes in a very readable, easy style. Commencing his book with the nature and history of geochemistry and a description of the earth in relation to the universe, discussing, along the way, the most acceptable theories of the origin of the solar system, he proceeds to a description of the structure and composition of the earth. Presenting the most plausible concepts of the three zones that comprise the earth—the crust, the mantle and the core—it is unusually interesting. Since rocks and minerals are the materials of which the crust and mantle are made and illustrate the principles of the structure of matter and of thermodynamics, it is appropriate that a brief summary of some thermodynamics and crystal chemistry be given. Following this, Dr. Mason describes the nature and formation of the three categories of rocks, igneous, sedimentary and metamorphic, and the geochemistry of the hydrosphere, the atmosphere and the biosphere.

The concluding chapter, the geochemistry cycle, is a summary of the whole science of geochemistry. Reading the book, one would become better oriented by beginning with this chapter. Brief but concise, it surveys the probable origin of the solar system, the formation of the earth and the subsequent development of its various components and the geochemical processes involved.

E. W. HAMMER, JR.

THEORETICAL PETROLOGY, by Thom F. W. Barth. 387 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.50.

One of the great enigmas of science is the physical processes in the origin, evolution, destruction, and redevelopment of the crustal formations of the earth.

In "Theoretical Petrology" Dr. Barth succeeds in bringing up-to-date our knowledge of the three principle rock formations—the igneous, sedimentary, and the metamorphic. Data

from many fields, geological, astronomical, chemical and physical are collated and interpreted to reflect the genesis and development of rocks.

To this reviewer, Part 1, dealing with the physics and chemistry of the earth as an astronomical body, was of particular interest. It supplements the newly published books of 1952 which deal with the origin and chemical composition of the earth. The other three sections are devoted to a comprehensive study of the three types of rocks and, as expected, by far the longest discussion is of the igneous rocks.

This book is most certainly not intended to be elementary and while of a highly technical nature it is destined to fill a need in the field. It demands of the reader a comprehensive knowledge and a complete grounding in the principles of the highly specialized science of petrology.

I. M. LEVITT

INTRODUCTION TO FACTORY PLANNING, By William Grant Ireson. 385 pages, illustrations, 15 × 22 cm. New York, Prentice-Hall, Inc., 1952. Price, $5.50.

The engineering approach to a problem—that is, where all data are collected, methodically assembled and logically analyzed—should be universally adopted. For the impossible problem frequently becomes possible; the difficult, easy; and the easy, easier when all the facts are gathered and the significant weeded from the insignificant; the solution often is obvious. It is just such an approach as this that Mr. Ireson, Acting Professor of Industrial Engineering at Stanford University, repeatedly recommends in all phases of factory planning. And this engineering method must be used in our modern, highly competitive industrial age if a manufacturer is to be successful. While an engineer may take such an approach to a problem for granted, he'd be surprised to learn how frequently hunches and preconceived ideas have been used as bases for decisions in past factory planning.

The factory that Mr. Ireson is concerned with is not the single brick building most people automatically think of when the term is mentioned; rather, it means any place where "the factors of production, land, labor, capital and enterprise are brought together for the creation of a good or service." It consists of all buildings, machinery, land, tools, materials, labor, parts, management, and services. All these factors are important in production and, fortunately, are considered in "Introduction to Factory Planning." Beginning with a clear discussion of factory planning, including types of industries, replanning old plants and building new ones, Mr. Ireson describes products and economic analysis including production sequence, market survey, methods of comparing monetary factors, labor, construction and repairs; principles of layout and departmentalization with comments on common layout errors, methods and procedures of planning for factory development; materials handling and materials handling equipment and applications; such factory services as electricity, steam and heat, power demand, water and piping systems; factory buildings and building features including old and new buildings, location of plants, design features and employee consideration; personnel facilities; and the management aspect of factory planning. The book is concluded with a number of problems in factory planning and layout—no doubt for classroom use—and appendices of compound interest tables, templates and models, and such data as materials handling and washroom information. The book is amply illustrated with photographs and sketches and contains many references at the end of each chapter.

The book is straightforward and clear, and, to one with an engineering training, much of the material may seem obvious. However, it should prove useful for, as has been indicated, Mr. Ireson has gathered a great deal of information on factory planning and has arranged and assembled it in an orderly, coherent fashion.

E. W. HAMMER, JR.

THE PHOSPHATIDES, by Harold Witcoff. 564 pages, 16 × 24 cm. New York, Reinhold Publishing Co., 1951. Price, $10.00.

Phosphatides represent a group of quite complex organic compounds that are found in animal organs and tissues, eggs, milk, vegetables and plants, bacteria, fungi, molds, and viruses. Biochemically and physiologically the phosphatides are involved in a great multiplicity of functions of great importance, about which very little is known. They are structural components of the living cell, and are important metabolic elements. Phosphatides also contribute

to various pathological functions and are of importance in serological medicine. In the living cell they form a complex chemical connecting link between the proteins and carbohydrates as structural units. Chemically they may be defined as esters of fatty acids with polyhydric alcohols that contain the phosphoric acid residue.

The first phosphatide was isolated in 1719, but it has been only within the last twenty years that the great bulk of work has been done on these very interesting compounds. In this book the author organizes and correlates the vast amount of information scattered throughout this field of science. He has made additions to existing information, corrected errors, and pointed out the gaps existing in the technology of phosphatides. The book is divided into six main sections: (1) the chemistry of phosphatides; (2) the analytical determination of the phosphatides; (3) phosphatides from plant sources; (4) phosphatides from animal sources; (5) biochemistry and physiology of the phosphatides; and (6) industrial aspects of phosphatides. At the end of each of these major sections is found an unusually complete bibliography covering the specific work found in each section.

While the field of phosphatides is extremely large and complicated, its many aspects are very well covered in Dr. Witcoff's book. The section of the book devoted to the biochemistry and physiology of the phosphatides is to be particularly noted because of its excellent organization of an enormous amount of material without the loss of continuity in presentation. This volume is to be highly recommended as a source book for both research workers and students in this branch of science. Since the emphasis in Dr. Witcoff's work is placed upon what is currently being done in phosphatide technology, this book becomes the most complete and up-to-date reference work in the field. DONALD H. RUSSELL

PROCEEDINGS OF A SECOND SYMPOSIUM ON LARGE SCALE DIGITAL CALCULATING MACHINERY, The Annals of the Computation Laboratory of Harvard University, Volume XXVI. 393 pages, illustrations, 14 × 28 cm. Cambridge, Harvard University Press, 1951. Price, $8.00.

The Annals of the Computation Laboratory of Harvard University are well known for their structure and content. Although this volume was not written (in whole) by the staff of the laboratory, its preparation was evidently carried out in accordance with standards normally associated with the series.

As stated in the preface by Howard H. Aiken, "Thirty-nine papers are herein published essentially as submitted."

Reviewing a book of this type is a delicate and difficult proposition, at best. Therefore, it behooves the commentator to restrict himself mainly to an exposition of the subject matter contained therein.

A brief explanation in passing for those unfamiliar with these symposia. They have consisted of conferences at which digital calculating machinery and techniques have been discussed. Opinions and descriptions of experimental work presented at these meetings have since been reduced to practice or found impracticable. In a sense, the symposia have served as a sounding board for scientists and engineers involved in the mechanization and simplification of computational routines.

For example, sixteen papers were devoted to reporting the most recent developments and futuristic trends in computing machinery. Seventeen more papers treated numerical methods, computational problems in physics, aeronautics, and applied mechanics. Furthermore, the commentator finds it gratifying that enough foresight existed to realize the effectiveness of utilizing digital computing machines to the economic and social sciences. Six papers related to these topics are included in this volume.

All in all, Volume XXVI should be of value to those interested in various aspects of digital computing machines and techniques, and applied mathematics. E. A. FASKO

SCIENCE SINCE 1500, by H. T. Pledge. 357 pages, illustrations, 16 × 25 cm. London, Ministry of Education; distribution in U. S. by The Philosophical Library, Inc.; 1951. Price, $5.00.

The staff of the Science Museum in London have established a sound reputation for writing excellent scholarly monographs on their special subjects. Mr. Pledge, who is the librarian at

the Museum, has contributed a work of a more general nature which will stand worthily among these publications. His book is an admirable achievement.

The history of science appears to be coming into its own, not only among scientists but also among layman, and this is cause for gratification. The young scientist commits an error if he neglects to study the history of his science and related sciences. For the student is given a simplified account of the discoveries of a Galileo, a Faraday, or a Harvey so that he misses the wonder in the achievement, and thinks the discovery so completely obvious that he could have made it himself. Knowing nothing of the contemporary state of knowledge or the equipment available, he fails to appreciate the master's accomplishment. Mr. Pledge furnishes an adequate background for the figures in his history, not failing to treat with the difficulties and failures they encountered. In most histories of science, the actors are seen against the intellectual and philosophical background of their times. Our author has chosen as his background the human and economic aspects of the successive periods and this gives a more realistic atmosphere to the story he has to tell.

A decided asset to the book is the inclusion of an adequate treatment of the part played by mathematics, notably as the foundation for nineteenth century physics. The layman may encounter some difficulty with these passages but they seem to have been inserted in such a manner that they can be skipped by the non-mathematician without detriment to the continuity.

Mr. Pledge's book should be read and appreciated by anyone interested in the origin of cardinal ideas in science. Solid and compact, it is, nonetheless, designed for continuous reading. Since this reviewer has repeatedly had cause to complain about the inadequacy of indexes in some recent works on science, he is happy to add a word of praise for the way in which this volume has been indexed, which will make it useful as a reference book.

T. C.

INTRODUCTION TO MODERN PRIME NUMBER THEORY by T. Estermann. 75 pages, 14 × 22 cm. New York, Cambridge University Press, 1952. Price, $2.50 (paper).

This little monograph was written to present some of the non-elementary results in number theory to mathematicians who do not specialize in this field. Only the very slightest acquaintance with the theory of numbers is assumed.

The following three results are considered: (1) A refinement of the prime number theorem, which states that the number of primes not exceeding m is asymptotic to $m/\log m$, is obtained by means of the Riemann zeta function. (2) A formula for the number of primes less than or equal to m in a given arithmetic progression is obtained. Methods of group theory are used in this problem, but the group theoretic results required are developed in the text. (3) Every odd number greater than a given sufficiently large positive constant has at least one representation as the sum of three primes.

A brief discussion of each result, and an outline of the method of proof is given prior to the presentation of the detailed proofs.

H. L. PLATZER

THE CITY, by Rod and Lisa Peattie. 108 pages, illustrations, 14 × 21 cm.
WATER FOR PEOPLE, by Sarah R. Riedman. 151 pages, illustrations, 14 × 21 cm.
PLANET EARTH, by Rose Wyler. 156 pages, illustrations, 14 × 21 cm.
New York, Henry Schuman, Inc., 1952. Price, $2.50 each.

These three are the first of a new "Man and his World" series geared for youngsters of the Junior High School age. The issuance of these by a publisher of the caliber of Henry Schuman insures a meaningful, well illustrated, exciting picture of a facet of natural and social science. The reader is reminded of the great contribution of this publisher to the history of science through his widely acclaimed "Life of Science" series.

In "The City" Rod and Lisa Peattie paint a graphic picture of the evolution of the city. They go back many thousands of years ago when there were no cities. From the primitive communities the evolution is traced to the first cities centered around the temples of worship. The story then shifts to the medieval city, the city of the machine in mid-nineteenth century

England and finally the modern city of today. Essentially, the book tries to convey to the youngster what is a city. It fulfills its mission.

Dr. Riedman, in "Water For People" has performed a magnificent job in taking a subject as prosaic as water and writing a fascinating story about it. The book deals with cosmology, paleontology, physics, meteorology, physiology, geology, and agronomy. It is difficult to pick up a glass of water, look at it, and imagine its role in each of the above sciences. Yet the author with a remarkably facile pen paints a detailed picture of water in many fields. As in the companion books of this series the book is correct and the information is up-to-the minute.

"Planet Earth" is a fascinating book which treats of all the earth. The book opens with the question of life and then shifts to earth's position as an astronomical body. In this latter section the author does a magnificent writing job. Also in this section occur some minor errors which it is hoped will be corrected in a latter edition. The time from new moon to new moon, i.e., the synodic month is 29½ days. In the distance to the star Sirius three zeros were lost. While the Andromeda star system is 750,000 light years away it is not the nearest. There are at least a half dozen others nearer. Substitution of "one of the nearest" for "the nearest" will take care of this.

The author was very wise in the arrangement of topics. She starts off with space, covers the atmosphere, then the surface and then goes below the surface. In doing this, her transitions are smooth and logical.

Included in this book are such current topics as space platforms and rain making.

"Planet Earth" is an admirable book in every respect.

I. M. LEVITT

ELECTROLYTIC MANGANESE AND ITS ALLOYS, by Reginald S. Dean. 257 pages, illustrations, 16 × 23 cm. New York, The Ronald Press, 1952. Price, $12.00.

Dr. Dean formerly was Chief Metallurgist and Assistant Director of the U. S. Bureau of Mines. Under his direction, in 1936, the Bureau developed a practical method of producing electrolytic manganese from domestic ore.

Manganese is the most plentiful nonferrous heavy metal, and can be the least costly. Electrolytic manganese can be used to supplement our supplies of copper, nickel and zinc in certain alloys. The scope of the book has been restricted to the production of electrolytic manganese, to the properties and uses of alloys in which it is necessary, and to alloys in which manganese produces properties significantly different from the use of other types of manganese. The book is divided into three main parts: production and properties of electrolytic manganese, nonferrous alloys of electrolytic manganese and ferrous alloys of electrolytic manganese.

In the opinion of the reviewer, the author has produced a masterpiece. He cites 340 references in this and in foreign countries. Of these, he is the author of 78. One hundred excellent illustrations show both the theory and the practice of the metallurgy involved in the production of the alloys, while 83 tables list the properties of the manganese ores and alloys. The book is recommended for anyone interested in the alloys of manganese, magnesium, zinc, aluminum, or special steels.

A. D. HAY

QUANTUM THEORY OF MATTER, by John C. Slater. 528 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $7.50.

The present volume by the author of several well-known texts on mechanics, electromagnetism and statistical mechanics which have appeared in the same series (International Series in Pure and Applied Physics), is addressed to a reader whose knowledge of mathematics and of the general subject of the structure of matter is at an intermediate graduate student level.

This book is to exhibit the reasoning and the mathematical methods used in the quantum-theoretical approach to problems of atomic structure and radiation and to illustrate the interpretation of observable physical properties of matter in the light of its results. The topics covered include: a brief review of the development of the theory culminating in Schrödinger's wave equation; examples of its exact solutions, and of approximate solutions obtained by perturbation methods; the hydrogen atom and molecule; atomic spectra; interatomic and

intermolecular forces; the metanic state; mechanical, chemical, thermal, electrical and magnetic properties of matter.

The material is well organized and is presented in the order of increasing complexity of the type of the physical systems considered. The associated mathematical techniques are exhibited on the simplest problems relating to a given type and their limitations and possible extensions to more complex problems are carefully discussed.

Special topics are relegated to 22 appendices occupying nearly 100 pages of the book. Problems given at the end of each chapter serve as exercises for the reader. At the end, a critical bibliography of books recommended for collateral reading is given.

This book is well-suited as a text for an introductory course on quantum mechanics for chemists, metallurgists, and workers in related fields, who require a fair understanding of the theory and of its results, but do not aspire to make it the principal object of their studies.

OTTO SPIES

BOOK NOTES

EXPERIMENTAL NUCLEONICS, by Ernst Bleuler and George J. Goldsmith. 393 pages, illustrations, plates, 15 × 23 cm. New York, Rinehardt & Co., Inc., 1952. Price, $6.50.

"Experimental Nucleonics," developed from a course at Purdue University, offers the fundamentals of the applications of nuclear physics to students planning to perform tracer research, as well as those students in straight nuclear physics. Designed to give a sound background in both chemical and physical aspects of the subject, the text includes 24 experiments over a wide range, so that different interests may be served. These experiments are designed to minimize health hazards and are largely limited to those not using accelerators. A prior knowledge of nuclear physics and chemistry is assumed. This new text should prove to be a valuable aid in the teaching of applied nuclear physics.

PRIMER OF ELECTRONICS AND RADIANT ENERGY, by Don Caverly. Second edition, 343 pages, 14 × 21 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $5.50.

As the title indicates, this is written for the non-scientist who would like to know more about such commonplace things as radio, television, X-rays, and other uses of electronics and radiant energy. Written in digest form, in as simple a language as possible in dealing with such subjects, the book covers basic principles of electricity, magnetism, electromagnetic radiation and electron tubes. This second edition includes many topics omitted from the first edition (1943) for security purposes. Although the book is aimed at salesmen, store clerks, housewives, doctors and lawyers, this reviewer questions whether such individuals will be able to understand many of the technical explanations. It should be quite useful as a reference work for high school libraries and science departments.

INTERMEDIATE COLLEGE MECHANICS, by Dan Edwin Christie. 454 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $7.00.

A new text in the field of college mechanics, this book stresses vectorial treatment of the subject. Planned as a second year course for physics or mathematics majors and engineers, it offers a thorough background of the language and algebra of vectors. While the symbolism used throughout the text deviates slightly from convention, the end result seems to be to avoid confusion between vectors and scalars, a difficulty encountered in writing such terms by hand rather than in print. Examples and exercises are provided for most of the sections, with answers given for the majority of the odd-numbered problems. The text is arranged so that whole sections or parts may be omitted, to fit the needs of varying abilities.

VADE-MECUM. Ninth edition, 444 pages, illustrations, 21 × 29 cm. Antwerp, P. H. Brans Ltd., 1952. Price, $5.00 (paper).

There is no need to do more than note the appearance of the ninth edition of this valuable guide for radiomen all over the world. In this edition only receiving and transmitting tubes

are listed, in order to keep the size and cost within reason. Also, many obsolete tubes have been omitted, making way for the newest types. All tubes are classified numerically and alphabetically in one single table, facilitating use of the book.

THE CHEMISTRY OF ORGANIC COMPOUNDS, by James Bryant Conant and Albert Harold Blatt. Fourth edition, 655 pages, 16 × 24 cm. New York, The Macmillan Company, 1952. Price, $5.90.

The fourth edition of this standard text has been published in order to bring up-to-date the fields of biochemistry and industrial chemistry. By eliminating out-dated material, the book has been kept to approximately the same size as the previous edition. A new feature in the present edition is the inclusion of basic information on the mechanisms of organic reactions and the directions which the reactions take; this presentation is limited to important general types and does not attempt to cover the subject thoroughly. A complete revision of the treatment of heterocyclic compounds adds to the usefulness of this edition.

GENERAL BIOLOGY, by James Watt Mavor. Fourth edition, 875 pages, diagrams, 16 × 24 cm. New York, The Macmillan Company, 1952. Price, $5.75.

In general, the plan of the fourth edition of this text for the beginning student in biology follows that of the previous editions, except that the section on elementary chemistry has been greatly reduced and placed in the appendix rather than in the main text. Among the chapters which have been rewritten are those on the cell, on coelenterates, and on flatworms; additions include material on the language of bees, new material on evolution, and a glossary of technical terms including pronunciation and etymology. Many new and revised illustrations help to clarify the presentation.

LABORATORY EXERCISES IN GENERAL BIOLOGY, by James Watt Mavor. Fourth edition, 333 pages, illustrations, plates, 21 × 28 cm. New York, The Macmillan Company, 1952. Price, $3.50.

Designed to accompany the fourth edition of the author's "General Biology," this laboratory manual offers a wide range of subjects, permitting its adaptation to the apparatus available in various laboratories. The manual is divided into two parts, Instructions and Work Sheets, the latter on heavy paper to facilitate drawing and perforated for easy removal. Sources of supply for needed materials are listed, as are the special materials required for each experiment.

INTRODUCTION TO ELECTRICAL ENGINEERING, by Robert Page Ward. Second edition, 412 pages, diagrams, 16 × 24 cm. New York, Prentice-Hall, Inc., 1952. Price, $5.75.

In the second edition of this text, first published in 1947, the only significant changes are found in the expansion of material relating to voltage notation, electric and magnetic field intensity, and flux density. Also, some 250 questions and problems have been added, while others have been reworded. The text is for an introductory, one-semester electrical engineering course, with adequate study questions covering each chapter.

MANUFACTURING PROCESSES, by Myron L. Begeman. Third edition, 608 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.00.

The third edition of the textbook has been revised to include the recent developments in processing, machines, machine tools and techniques employed in foundry practice, pattern work, plastic molding, and cold forming of metals. The latter half of the book covers measuring instruments, cutting tools, machines and their accessories. The information is fundamental in nature and should be found useful as reference reading for engineers engaged in production work.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

***Abstract of* Alteration of the Mortality of Roentgen Irradiated Mice by Chemical Means.**—G. LIMPEROS (*Am. J. Roent., Rad. Ther. Nuclear Med.* **67,** 810 (1952)). The mortality of roentgen irradiated mice was found to be altered by chemical agents capable of influencing the generation of free radicals from cellular water such as sulfhydryl compounds, sulfhydryl inhibitors, and oxygen. Thiourea (2250 mg./kg.) injected intraperitoneally before roentgen irradiation was found to decrease the mortality of mice irradiated with 450 r, 500 r, and 600 r. This protective effect was enhanced when the thiourea dose was increased up to 4000 mg./kg. Thiourea injected after irradiation failed to protect mice against irradiation. The protective effect of thiourea was diminished when folic acid was injected at the same time as thiourea and also administered in the drinking water after irradiation. Thiouracil, although it contains a free sulfhydryl group, did not protect mice when injected intraperitoneally before irradiation. Sodium maleate injected intraperitoneally before irradiation increased the mortality of mice, possibly because the sulfhydryl-containing constituents of the tissues were lowered by interaction with sodium maleate.

A 10 per cent oxygen concentration during irradiation was found to decrease the mortality of roentgen-irradiated mice, whereas a 100 per cent oxygen concentration was found to increase mortality although not to a significant degree. When the oxygen concentration of the atmosphere was changed after irradiation, the mortality was not changed significantly.

The results are discussed in relation to the indirect mechanism theory for the action of roentgen irradiation *in vivo*. Their possible significance in radiation therapy is also discussed.

CURRENT TOPICS

150th Anniversary of du Pont Company.—The 150th anniversary of the establishment of E. I. du Pont de Nemours & Company will be marked July 18 at the site of the first du Pont powder mills on the banks of Brandywine Creek near Wilmington. The ceremonies, which will include a simple historical dramatic prologue, addresses by company officials, and the dedication of a marker on the site of the first plant, are the focal point of observances by the du Pont Company.

About 6000 spectators—representatives from the 71 plants and Wilmington offices, retired employees, members of the du Pont family, and guests—will be on hand for the program. Thousands of men and women at du Pont plants, laboratories and sales offices, as well as their families and friends, across the country can hear the program which will be broadcast over a national network.

The site of the July 18 program is on the property of Mrs. Francis B. Crowninshield, a direct descendant of Eleuthere Irenee du Pont, founder of the company. She lives in the house that he began building in 1802, and into which he and his wife, Sophie, and their children, moved in 1803.

The ground floor of Mrs. Crowninshield's home, as well as the company's first office building, and the small stone house where the founder and his wife lived during their first summer in Delaware, will be open to visitors immediately preceding the anniversary ceremonies.

Spectators will sit in a natural amphitheatre facing the Brandywine. Trees and shrubs that would have blocked the audience's view of the proceedings have been transplanted as part of a landscaping program begun last year and now nearly completed. The drive along the Brandywine and site of the old mills will be open to the public for several weeks following the ceremony.

The ceremony is being held one day in advance of the actual anniversary date, as it was on July 19, 1802, that E. I. du Pont arrived on the Brandywine to begin construction of his first mill. The shift was made to avoid conflict with weekend plans of those attending the ceremony.

William A. Hart, director of the du Pont Advertising Department and chairman of the anniversary committee, explained that the occasion marks the dynamic growth of a small mill filling the single, though paramount, need of a young nation for blasting and gun powder, into an organization employing 87,000 men and women, owned by 138,000 stockholders, selling thousands of chemical products to more than 75,000 customers, and buying from more than 30,000 suppliers.

Man-Made Weather.—Pioneering work of General Electric Company scientists in weather modification "will eventually have a profound effect on all of us," Dr. C. G. Suits, G-E vice president and director of research, predicted recently.

His comment followed a recent announcement of the termination, on June 30, of Project Cirrus, joint weather research program of the Signal Corps and Office of Naval Research in consultation with the G-E Research Laboratory.

The program has been carried out for the past five years by scientists of the laboratory under a contract sponsored by the two services. Aircraft and crews needed in some experiments have been furnished by the Air Force.

Early exploratory phases of cloud seeding and allied research concerned with the physics of clouds, called for under Project Cirrus, are now virtually complete, Dr. Suits said.

According to the G-E official, "the work of Dr. Irving Langmuir, Dr. Vincent J. Schaefer, and Dr. Bernard Vonnegut in this field has initiated a new science of experimental meteorology and has provided a tremendous stimulus to scientific investigations in this most important subject.

"It is not easy to predict ultimate results," he said, "but it is certain that the pioneering and spectacular work of these scientists in cloud physics, cloud seeding, and weather modification will eventually have a profound influence on all of us. Rainmaking, which has been popularly identified with this project, is in reality only one aspect of the very broad subject of experimental meteorology."

Wide interest in experimental weather study has been aroused by work of the project, according to Dr. Suits. So many other research projects have been stimulated by the pioneering efforts of the scientists working under Project Cirrus that continued progress in the search for new basic knowledge of weather phenomena "seems assured," he said.

Active research in this field is being carried on in many foreign countries. Among these, he said, are Canada, Cuba, Peru, England, France, Switzerland, Israel, Algeria, Tanganyika, Union of South Africa, Formosa, Japan, and Australia.

According to current estimates, some 350 million acres in the United States west of the Mississippi are subject to cloud seeding by commercial operators, he added.

Because of the widespread interest in experimental meteorology, more emphasis is now being placed on consulting activities by Dr. Langmuir and Dr. Schaefer, according to the G-E official. In addition, there are many problems connected with cloud physics which are being studied by Dr. Schaefer. It is expected that this laboratory work in cloud physics will be continued, in order to learn more about the fundamental nature of the phenomena involved."

The program leading to Project Cirrus began in the summer of 1946. Dr. Schaefer, who had long been interested in snowflakes as a hobby and who had, during World War II, worked on icing of airplanes, sought to find what caused snow crystals to form in the atmosphere.

Inside a home freezer, at a temperature of about zero F., he made a cloud of tiny water droplets, which did not freeze despite the low temperature, well below the commonly-accepted freezing point of water at 32 degrees F.

Although he tried adding many materials he could not get snow crystals to form. Then one hot July day, in order to keep the box sufficiently cool, he introduced some dry ice. The cloud was immediately converted into one of minute ice crystals.

According to Dr. Schaefer, it is not the dry ice itself but rather its cooling effect that causes the transformation from water droplets to ice crystals.

On Nov. 13, from an airplane flying at 14,000 feet over Mt. Greylock in western Massachusetts, Dr. Schaefer applied this discovery out-of-doors. He

dropped dry ice pellets on a natural supercooled cloud and saw it turn to snow. None reached the ground, however, because it melted and evaporated as it passed through warmer layers some 3000 feet below.

A little later, Dr. Bernard Vonnegut, another G-E scientist, found that crystals of silver iodide, which have the same arrangement of atoms as ice crystals, act as nuclei in clouds and cause supercooled water droplets to freeze on them. All these experiments were guided by the theoretical work of Dr. Irving Langmuir.

A third experimental technique for modifying cloud formation is that of water seeding developed by Dr. Langmuir. Water drops, introduced into certain types of cumulus clouds, grow and break up in a continuous chain reaction, which effect serves to reorganize tiny water droplets composing the cloud into raindrops large enough to fall. This technique recently was reported used very successfully in Australia by the government of that country.

Project Cirrus was established in March, 1947. Actual flying and seeding was done by an operations group, consisting entirely of men from the services, while the General Electric scientists analyzed the data and advised on field work.

New "Plastic Kidney."—A new-type "plastic kidney" has been developed by Westinghouse Electric Corporation engineers who were guided in their efforts by members of the medical profession. The new "kidney"—one of the most compact and mobile units yet built—is now in successful operation at two of the nation's leading hospitals.

Dr. T. S. Danowski, senior staff physician at Children's Hospital, Pittsburgh, Pa., revealed that a clinical model of the improved "kidney" has performed outstandingly in its application to adult patients over a period of several months. A similar unit is in use at Peter Bent Brigham Hospital in Boston, Mass., whose staff cooperated in the development.

The improved kidney was designed and developed by George W. Jernstedt, manager of the Westinghouse Special Products Division Engineering Department, and his associate Harry H. Barker. Enclosed in a metal cabinet, the complete unit and accessories can be transported quickly from hospital to hospital with the aid of one or two technicians.

Key feature of the unit is a 50-ft. length of thin flexible cellophane tubing wound spirally around three concentric hollow-cylinders made of expanded metal screen. As blood is drawn from an artery in the patient's arm, it passes through the tubing and in so doing is purified as impurities filter out through microscopically small holes in the tubing: holes that are too small for larger blood components, such as corpuscles, to leak out.

The metal screen cylinders and the encircling plastic tubing are immersed in a specially prepared bath called a "dialyzate," and as the impurities diffuse out of the tubing, they pass into the bath and are drained off. The purified blood continues through the tubing and returns to the patient by way of a venous opening in the other arm. An electric-eye safety device immediately shuts off the unit if a leak appears in the tubing or if a blood-clot occurs.

To make sure that the patient loses no vital blood components while impurities are being removed, the dialyzate bath is made up of compounds such as sodium, potassium, and carbonate in just the right proportions. By varying

the pressure between the blood and the bath, and by keeping the right concentration of compounds, an equilibrium is set up and normal blood composition is maintained during the removal of impurities.

The new "plastic kidney" offers several advantages over previous types of artificial kidneys.

The ease with which it can be moved from one hospital to another makes it highly useful in emergencies. In addition, it is the first unit to be designed in which complete sterilization can be performed with the components in operating position, thus eliminating long delays in dismantling and reassembling. The new "kidney" contains no rotating joints, valves, or excessive points of pressure. This reduces the danger of blood clotting or breaking of blood cells to virtually zero, and makes for easier, longer operation of the device.

The "kidney" has been successfully used for combatting chronic and acute uremic poisoning that takes place when kidney functioning breaks down. In cases of poisoning caused by sulfa drugs, barbiturates, or carbon tetrachloride, it is believed the "kidney" can take over the job of the overworked human organ. It may also prove useful after certain abdominal operations, when kidney blockage results, or to reduce excessive amounts of such substances as potassium in the blood. In research the unit is a valuable tool in the study of body functions.

Westinghouse does not contemplate further work on the kidney project, Mr. Jernstedt indicated, nor does the Company at this time anticipate commercial activities in this field. The firm will make information available, however, to qualified organizations.

Lens Attachment Eliminates Retouching.—A new item of unusual interest to professional photographers and advanced amateurs is the Pictrol, announced by Craftsmen's Guild of Hollywood, California. Pictrol is a low-priced lens attachment which automatically enables anyone to achieve results formerly obtained only through the use of high-priced, variable diffuse image lenses. Used in conjunction with any lens, Pictrol converts it into a variable soft focus lens, which gives any degree of diffusion from razor-sharp to misty soft. Attached to an enlarger, Pictrol eliminates retouching as well as spotting. It reduces grain, softens harsh contrasts, and imparts to the print a professional quality of pictorial softness. Diffusion may be accurately controlled simply by turning the calibrated ring. Identical prints may be obtained repeatedly. Pictrol is simple to attach, easy to use and fits all lenses.

New Device for Measuring Moisture Content of Commodities.—Substantial time savings are now possible in determining the moisture content of a wide variety of commodities, ranging from cereal grains to foundry sands and detergents, through a new instrument produced by Central Scientific Co., Chicago.

Called a moisture balance and employing a new principle in drying samples by infrared radiation, the device precisely detects the moisture content of a commodity in five to a maximum of fifteen minutes. Older methods of drying samples in ovens to measure their moisture loss require one to twelve hours to obtain the same result.

Mass production processes where time is an essential factor represent one of the more important industrial and commercial applications of the new mois-

ture balance. The economies effected by the rapidity of the drying process are another industrial consideration.

A highly sensitive torsion balance is the heart of the device. The operation of the instrument is as follows: first, five grams of the commodity to be tested are placed in a light aluminum pan inside the unit, then the infrared radiation is turned on, and, finally, the weight of the moisture loss is shown by the torsion balance on a scale calibrated in percentage points.

Fluorescent Floodlights.—Giant fluorescent floodlights which produce virtually glareless light are now being used at an airport for the first time.

Eight-foot-long tubular fluorescents recently were installed on the roof edge of the American Airlines loading section at Logan International Airport's terminal building in Boston.

The installation is part of a joint experiment by American Airlines and General Electric Company engineers to determine the practicability of fluorescent floodlighting for passenger loading ramps and maintenance areas.

According to J. H. Woods, manager of operations for American Airlines, "the new fluorescents have made our loading platform here one of the best-lighted in the country." Although the new lighting installation provides a substantial increase in the amount of illumination in the area, the power consumed by the fluorescents is less than previously required.

A total of eight G-E aluminum fixtures, bracketed to the hand rail of the observation deck 15-feet above the ground, comprise the installation. Each fixture contains two 8-foot-long fluorescent lamps. A highly-polished, scientifically-designed reflector directs light from the lamps through clear plastic windows to the loading area.

Mr. Woods said that American Airlines is the largest user of Logan airport's terminal, routing 80 flights per day. The new lights, he said, have enabled American Airlines to handle night passenger loading and maintenance work faster and more efficiently.

He stated that pilots, loading crews, and passengers say that the fluorescents are far more comfortable on the eyes than conventional incandescent floodlights.

Electronic "Potting" Materials.—A new group of room-temperature cure plastic casting materials developed by the B. G. Forman Company, Inc., of Brooklyn, for "potting" and "encapsulating" transformers and electronic assemblies has aroused armed services interest.

Encapsulating or "embedding" of complete electronic units was tried experimentally during World War II. Replacement and repair of signal and other electrical equipment under battle conditions proved to be a serious handicap to communications. Out of this problem a new philosophy has emerged: Complete "throwaway" of electronic units embedded in solid plastic blocks which protect the components from shock, moisture, and vibration. Conventional casting resins, however, shrink to such a great extent during the setting process, that the metallic or glass components fracture or are thrown out of place. In addition, setting requires from 24 to 36 hours, with complicated baking operations.

The new Forman materials, Polyform G series, have overcome all of these disadvantages. Ease of casting and rapidity of cure are their outstanding

production advantages. They can be set at room temperature in 5 to 15 minutes. The shrinkage problem has been eliminated; units can be handled at high production rates by unskilled workmen without fear of damage. In addition, the materials absorb no moisture from the surrounding atmosphere, guaranteeing that the electrical characteristics of the embedded equipment will remain unimpaired.

The Polyform G series consists of three formulas: No. 107, slight shrinkage, for ease of mold extraction; No. 622, no shrinkage, generally used for potting transformers in cans; and No. 1230, slight expansion, used to embed complete units and adhere to walls of cavity.

According to Mr. G. G. Forman, research director and specialist in metal-to-plastic combinations, the new materials take the potting process out of the "mudpie" stage and into the realm of competitive mass production. Utilizing casting methods made possible with these materials, components of varying size from subminiature radio tubes up to large bus bar units can be successfully encapsulated.

Because viscosity of the new materials can be adjusted over a wide range, from waterlike up to a thick tarry syrup, the entire electrical industry is interested. In small transformers the new materials replace conventional potting materials including varnishes and waxes. Used with vacuum, a low viscosity formulation penetrates every part of the transformer thoroughly, and achieves complete hermetical sealing.

Army Rocket Research fights TB by voluntarily deferring delivery of hydrazine, a chemical compound showing promise as an improved rocket propellant. Hydrazine is also an essential part of a new family of anti-tuberculosis drugs which show encouraging results in treatment of tubercular patients. Only large scale manufacture of hydrazine has been sponsored by Army Ordnance Corps; release of the compound into commercial channels will assure continuation of clinical tests. Although many TB cases which appeared to be hopeless have shown improvement after treatment, the drugs are not yet considered a cure.

A new type steam catapult will be installed on the aircraft carrier *Hancock* in 1954, with enough power to sling a jet fighter into the air even when the ship is headed downwind. Developed by the Royal Navy, the catapult uses a slotted cylinder through which a piston is driven by high pressure steam from the ship's boilers. A hook on the aircraft to be launched is connected to the piston; a novel sealing device keeps steam from escaping through the cylinder slot. The Navy conducted tests with a "steam slingshot" aboard H.M.S. *Perseus* during past three months, is investigating possibility of using it in the new *Forrestal*. No details have been released.

A new type of Diesel locomotive built to Army Transportation Corps specifications by GM, can run on tracks of varying gauge, from U. S. standard up to the widest in use. Simple mechanical adjustment of wheels on the axles permits change from one gauge to another. The engine can also be fitted with couplers of any type used. Power plant is a sixteen cylinder, two-cycle Diesel rated at 1600 hp., giving top speed of 77 mph. Design permits operation in weather extremes of minus 40° F. to plus 125° F., allows use of 40 cetane fuel oil or lower. Thirteen MRS-1 locos will be delivered.